## NEW INDIRECT SYLLOGISTIC MOODS

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**Abstract:** In this paper are presented the indirect syllogisms that are divided in two categories. These syllogisms are resulting from operation applied either to the premisses, being named primary indirect syllogisms, or to the conclusion of other syllogisms, being named secondary indirect syllogisms. For the first category three new primary indirect syllogisms have been established by us, by one in the second, third and fourth figures. For the second category thirteen indirect syllogisms are proposed. So, in the total, twenty-four indirect syllogisms are resulted, by six in every figure. **Keywords**: indirect syllogism, mood, conversion, subalternation

# 1. Introduction

Indirect syllogisms are those whose conclusion states the major term depending on the minor term, so that the major term is the subject and the minor term is the predicate. According to the way in which the conclusion is got, they are:

• primary or genuine indirect syllogisms, whose conclusion results from the operations applied only to the premisses;

• indirect syllogisms by conversion, the conclusion of which is obtained by conversion of the conclusion of the direct syllogisms;

• indirect syllogisms by subalternation, the conclusion of which is resulting by subalternation to the conclusion of indirect syllogisms with universal conclusion.

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Indirect syllogisms by conversion and subalternation are secondary syllogisms, because they are obtained by operations applied to the conclusions of other syllogisms.

#### 2. Establishing of new indirect syllogistic moods

The syllogisms of the first category result from certain combinations of premisses. Some combinations are indicated by Aristotle himself at the beginning of the chapter 7 of the first part of *Prior Analytics*:

It is obvious [...] that, in all the figures, whenever there is no syllogism, if both terms are affirmative or negative, nothing is necessary; but if one is affirmative and the other negative, and if the negative is taken universally, there will always be a syllogism relating the minor term to the major: for example, if A belongs to all or some B, and B does not belong to any C, [...] it is necessary that C does not belong to some A. (*An. pr.*, I, 7, 29a, 28-36).

Thus, "in all the figures", if the premisses are according to the text, primary indirect syllogisms will result. The premisses of these syllogisms are specified by the example given: the major premiss must be affirmative, universal or particular, and the minor premiss must be negative universal.

Another combination of premisses from which primary indirect syllogisms are resulting were established by us; it will be presented, in the paper, at the right place.

Following the indications given in the quoted text, Theophrastus (Dumitriu 1969, 189) established, for the first figure, two indirect syllogistic moods that were named by scholastic *Fapesmo*, the one with the universal-affirmative major premiss and the universal-negative minor premiss, and *Frisesomorum*, the one with the particular-affirmative major premiss and the universal-negative minor premiss (Didilescu and Botezatu 1976, 100); the conclusion of these moods is "Some P is not S".

For the second figure Iulius Pacius (1550-1635) established, also based on the quoted text, the primary indirect syllogistic mood *Firesmo*,

with the particular-affirmative major premiss and the universal-negative minor premiss and the conclusion "Some P is not S". The second primary indirect syllogism of this figure, with the universal-affirmative major premiss and the universal-negative minor premiss, is not recognized, because its conclusion resulting from the operations applied to these premisses and stating the major term depending on the minor term, was converted giving the direct syllogistic mood *Camestres*. The conversion of the conclusion was made by Aristotle himself, as it appears from the text:

[...] if M [the middle] belongs to the whole N [the major], but not to O [the minor], then N will not belong to any O. For if M does not belong to any O, neither O belongs to any M; but M (as has been said) belongs to all N; then O will not belong to any N because the first figure has been made again. But since the negative relation is convertible, N will not belong to any O. (*An. pr.*, I, 5, 27a, 11-17)

However, as Aristotle says, two chapters later, that "in all the figures" the syllogisms that fulfil the conditions mentioned in the first quoted text, report "the minor term to the major", he implicitly recognizes the existence of the primary indirect syllogism of the second figure with the conclusion "No P is S", which can be called *Cameste* and from which the mood *Camestres* is resulting by conversion of the conclusion. The idea of difference between the moods *Camestres* and *Cameste* also arises from Lukasiewicz's following text (1958, 27): "It is important that propositions of the type 'A belongs to no B' and 'B belongs to no A' are regarded by Aristotle as different".

Hence "All S is P" is different from "All P is S", what justifies the mood *Cameste*. Consequently, the following are highlighted: 1) the second figure also fulfils the conditions of the Aristotelian text; 2) the direct mood *Camestres* derives from an indirect mood by conversion of its conclusion.

The combination of premisses that leads to the primary indirect syllogisms established by us consists of a particular-negative premiss and a universal-affirmative one; it is applicable only to the second and third figures. For the second figure, the primary indirect syllogism with this combination of premisses has the particular-negative major premiss and the universal-affirmative minor premiss:

The proof of the conclusion is made by the indirect method of *reductio ad impossibile;* its contradictory is the sentence "All P is S" and replaces the major premisse in the mentioned syllogism, so that are obtained the premisses of the mood *Bramantip*:

By transposing these premisses the mood *Barbara* is achieved:

the conclusion of which is contradictory to the major premiss of the initial syllogism (1), so "Some P is not S" is the conclusion of the syllogism (1). The proposed name for the syllogism analyzed is *Brocamo*, where: b shows that the syllogism is reduced to the mood *Barbara*; group br specifies that it is obtained the mood *Bramantip*, as an intermediate syllogism; c indicates the replacement of the major premiss with the contradictory of the conclusion; m indicates the permutation of the minor premiss with the major one.

To illustrate the deductive process of *reductio ad impossibile* used to prove the conclusion of the syllogism (1), we propose the diagram in figure 1.

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The symbols used have the following significations:

'A', 'O', are constants and have the known meaning;

'p' – major term, 'm' – middle term, 's' – minor term;

curly braces, '{}', delimit the contradictory of a sentence;

the double arrow,  $\Longrightarrow$ , shows the sense of the contradictory transformation;

the simple arrow, ' $\rightarrow$ ', indicates the sequence of deductive operations.

Tr indicates a transposition of premisses

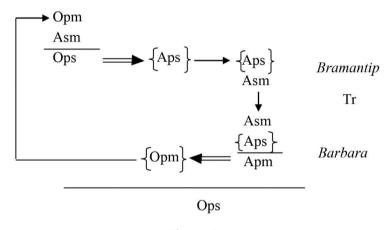


Figure 1

The graphical representation proposed for the deductive process of *reductio ad impossibile* is suggestive and has the following advantages over that used by Clark (1980, 9-11): 1) clarity of the demonstration operations and their sequence; 2) highlighting the initial syllogism as the starting and closing point of the deductive process; 3) vertical writing of the syllogism premisses facilitates the indication of the performed operations. The closing of the deductive process discharges the assumption that the conclusion of the initial syllogism is the contradictory to its conclusion and proves this conclusion. With this indirect syllogistic mood, the second syllogistic figure is enriched with another primary indirect syllogistic mood. The deductive process represented in the figure 1 can be written in the natural deduction form. The symbols used, besides the precedents, are:

square brackets '[',']', for the numbers of premisses;

'~' – sign of negation;

'Cs' – conclusion;

'Cd' - contradictory;

r.a.i. – reductio ad impossibile.

The rules of inference used are the moods Bramantip and Barbara.

For an exact preservation of the sequence of the deductive process operations represented diagrammatically, it is necessary to repeat some lines, what is indicated by the word "repetition".

2 Cs
Cd
, repetition
5 Bramantip
5 Tr
7 Barbara
7 Cs
Cs
Cd
, r.a.i.
2 Cs

In the first column on the left side are written only the premisses and their negation.

The deductive process can, also, be written using the rules of ~elimination and ~introduction of the Gentzen's system of natural deduction, because it corresponds to *reductio ad impossibile*. In this system the proof can be done both in the propositional logic and in the monadic predicate logic. In the propositional logic the proof is:

1 (1) Opm	Pr
2 (2) Asm	Pr
1,2 (3) Ops	1,2 Cs
4 (4) ~ Ops	assumption
4 (5) Aps	4 Cd
2,4 (6) Apm	2,5 Cs Barbara
2,4 (7) ~ Opm	6 Cd
1,2,4 (8) ^	1,7 ~E
1,2 (9) ~ ~ Ops	4,8 ~I
1,2 (10) Ops	9 DN

This writing is more compact then first, but it does not illustrate all deductive process.

In monadic predicate logic the proof of the syllogism (1) is:

$(\exists x) (Px \bullet \sim Mx), (\forall x) (Sx \rightarrow Mx) \vdash (\exists x)$	(Px∙~Sx)
1 (1) (∃x) (Px∙~Mx)	Pr
2 (2) (∀x) (Sx→Mx)	Pr
3 (3) ~(∃x) (Px•~Sx)	assumption
3 (4) $(\forall x) \sim (Px \bullet \sim Sx)$	3 QS
3 (5) (∀x) (Px→Sx)	4 DeM
3 (6) Pa→Sa	5 <b>∀</b> E
2 (7) Sa→Ma	2 ∀E
8 (8) Pa	assumption
3,8 (9) Sa	$6,8 \rightarrow E (MP)$
2,3,8 (10) Ma	$7{,}9\rightarrow {\rm E}~({\rm MP})$
2,3 (11) Pa→Ma	$8,10 \rightarrow \mathrm{I}$
2,3 (12) (∀x) (Px→Mx)	11 ∀I
2,3 (13) ~(∃x) ~(Px→Mx)	12 QS
2,3 (14) ~(∃x) (Px•~Mx)	13 DeM
1,2,3 (15) ^	1,14 ~E
1,2 (16) ~ ~ ( $\exists x$ ) ( $Px \bullet ~Sx$ )	3,15 ~I
1,2 (17) (∃x) (Px∙~Sx)	16 DN

The Gentzen's system of monadic predicate logic allows, also, the use of direct method for proving the syllogisms (1) and (4). For the syllogism (1) the direct proof is:

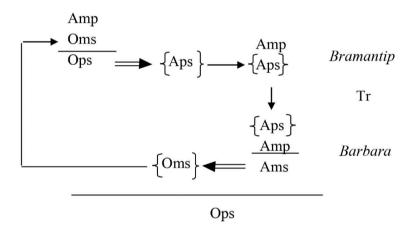
$(\exists x) (Px \bullet \sim Mx), (\forall x)(Sx \rightarrow Mx) \vdash$	- (∃x) (Px∙~Sx)
1 (1) $(\exists x) (Px \bullet \sim Mx)$	Pr
2 (2) $(\forall x)$ (Sx $\rightarrow$ Mx)	Pr
3 (3) Pa∙~Ma	assumption
2 (4) Sa→Ma	2 ∀E
3 (5) ~Ma	3 ●E
2,3 (6) ~Sa	4,5 MT
3 (7) Pa	3 ●E
2,3 (8) Pa∙~Sa	6,7 ●I
2,3 (9) (∃x) (Px•~Sx)	8 ∃I
1,2 (10) $(\exists x) (Px \bullet \sim Sx)$	1,3,9 ∃E

For the third figure, Iulius Pacius has established, from the same indications of Aristotle, the primary indirect moods *Fapemo* and *Frisemo* (Didilescu and Botezatu 1976, 100); the conclusion of these syllogisms is "Some P is not S". In the case of this figure the combination of premisses established by us is composed of the universal-affirmative major premiss and the particular-negative minor premiss:

and gives the third primary indirect syllogistic mood of this figure. The proof of the conclusion is made, also, by *reductio ad impossibile*. The contradictory of the accepted conclusion is "All P is S" and it will replace the minor premiss in the initial syllogism (4). Thus, it is obtained the mood *Bramantip*:

Transposing the premisses the mood *Barbara* is got:

Its conclusion is the contradictory of the minor premiss of the initial indirect syllogism (4). Hence, the conclusion established for the initial syllogism (4) is correct. The proposed name for the syllogism (4) is *Bramoco*, where the letters have the same meaning as in the previous case. The graphical representation of the proof of the conclusion of this syllogism by *reductio ad impossibile* is given in figure 2.





The natural deductive form of the representation of the figure 2 is:

[1] (1) Amp	Pr
[2] (2) Oms	Pr
[1,2] (3) Ops	1,2 Cs
~ [1,2] (4) Aps	3 Cd
[1] (5) Amp	Pr, repetition
~ [1,2] (6) Aps	3 Cd, repetition
	5,6 Bramantip

5,6 Tr
7,8 Barbara
7,8 Cs
9 Cs
10 Cd
Pr., r.a.i.
1,2 Cs

Like above, the deductive process of the figure 2 can be written in the Gentzen's, system of natural deduction for propositional logic too.

1 (1) Amp	Pr
2 (2) Oms	Pr
1,2 (3) Ops	1,2 Cs
4 (4) ~ Ops	assumption
4 (5) Aps	4 Cd
1,4 (6) Ams	5,1 Cs Barbara
1,4 (7) ~ Oms	6 Cd
1,2,4 (8) ^	2,7 ~E
1,2 (9) ~ ~ Ops	4,8 ~I
1,2 (10) Ops	9 DN

The proof in the monadic predicate logic of the syllogism (4) is:

$(\forall x) (Mx \rightarrow Px), (\exists x) (Mx \bullet \sim Sx) \vdash$	$(\exists x) (Px \bullet \sim Sx)$
$1(1)(\forall x)(Mx \rightarrow Px)$	Pr
2 (2) (∃x) (Mx∙~Sx)	Pr
3 (3) ~(∃x) (Px•~Sx)	assumption
3 (4) $(\forall x) \sim (Px \bullet \sim Sx)$	3 QS
$3(5)(\forall x)(Px \rightarrow Sx)$	4 DeM
3 (6) Pa→Sa	5 ¥E
1 (7) Ma→Pa	1 ¥E
8 (8) Ma	assumption
1,8 (9) Pa	$7,8 \rightarrow E (MP)$
1,3,8 (10) Sa	$6,9 \rightarrow E (MP)$
1,3 (11) Ma→Sa	$8,10 \rightarrow I$

1,3 (12) (∀x) (Mx→Sx)	11 ∀I
1,3 (13) ~(∃x) ~(Mx→Sx)	12 QS
1,3 (14) ~(∃x) (Mx∙~Sx)	13 DeM
1,2,3 (15) ^	2,14 ~E
1,2 (16) ~ ~ (∃x) ( $Px \bullet ~Sx$ )	3,15 ~I
1,2 (17) $(\exists x) (Px \bullet \sim Sx)$	16 DN

The direct proof of the syllogism (4), using Gentzen's system, is:

$(\forall x) (Mx \rightarrow Px), (\exists x) (Mx \bullet \sim Sx) \vdash$	$(\exists x) (Px \bullet \sim Sx)$
$1(1)(\forall x)(Mx \rightarrow Px)$	Pr
2 (2) (∃x) (Mx∙~Sx)	Pr
1 (3) Ma→Pa	1 ∀E
4 (4) Ma∙~Sa	assumption
4 (5) Ma	4 ●E
1,4 (6) Pa	$3,5 \rightarrow E (MP)$
4 (7) ~Sa	4 ●E
1,4 (8) Pa∙~Sa	6,7 ●I
1,4 (9) $(\exists x) (Px \bullet \sim Sx)$	8 ∃I
1,2 (10) (∃x) (Px•~Sx)	2,4,9 ∃E

Another primary indirect syllogistic mood of the third figure can be considered the one whose conclusion "Some P is S" was converted for obtaining the direct mood *Disamis*. This syllogism is:

As a name for this syllogism is proposed *Disami*, which highlights the fact that the mood *Disamis* was got by conversion of the conclusion of the syllogism (7). With this syllogism, the third figure has four primary indirect syllogisms.

Between the new-established primary indirect moods, *Brocamo*, for the second figure, and *Bramoco*, for the third figure, and the direct moods *Baroco* and *Bocardo* of the two figures, there is an analogy given by the following characteristics:

a) all have a particular-negative premiss and the other universal-affirmative, a combination of premisses which can only be found in the second and third figures, where the middle term has the same function in both premisses;

b) for the same figure the major premiss of the direct mood becomes the minor premiss in the indirect mood and vice versa, the minor premiss of the direct mood becomes the major premiss in the indirect mood;

c) the direct mood of the second figure becomes indirect mood in the third figure, and the direct mood of the third figure becomes indirect mood in the second figure, respecting the position of the middle term of each figure;

d) the conclusion, both of the direct and the indirect moods, is proved by *reductio ad impossibile* which reduces them all to the *Barbara* mood. The indirect moods are reduced to *Barbara* by means of the mood *Bramantip*.

For the fourth figure, we have established the following primary indirect syllogism, which fulfils the conditions given by Aristotle:

By transposing the premisses one obtains the mood Ferio:

As a name for the syllogism (8) it is proposed *Fimeno*, where the letters i, e, m and o have the known significations and n helps to pronounce the word.

The syllogism (8) can be, also, proved by Gentzen's system of natural deduction, in the monadic predicate logic, but using only the direct method.

$(\exists x) (Px \bullet Mx), (\forall x) (Mx \rightarrow \sim S)$	$x$ ) $\vdash$ ( $\exists x$ ) ( $Px \bullet \sim Sx$ )
1 (1) $(\exists x) (Px \bullet Mx)$	Pr
2 (2) (∀x) (Mx→~Sx)	Pr
3 (3) Pa•Ma	assumption
2 (4) Ma→~Sa	2 ∀E
3 (5) Ma	3 ●E
3 (6) Pa	3 ●E
2,3 (7) ~Sa	4,5 MT
2,3 (8) Pa∙~Sa	6,7 ∙I
2,3 (9) (∃x) (Px•~Sx)	8 ∃I
1,2 (10) (∃x) ( $Px \bullet \sim Sx$ )	1,3,9 ∃E

The second indirect syllogistic mood specified in Aristotle's text – with the universal-affirmative major premiss and the universal-negative minor premiss was used, as in the case of the second figure, for obtaining the direct mood *Camenes*. Since the conclusion of this mood is the converse of the conclusion that states the major term depending on the minor term resulting from the mood *Celarent* to which the syllogism with the mentioned premisses is reduced, the syllogism:

$$\begin{array}{c}
\text{All P is M} \\
\text{No M is S} \\
\hline
\text{No P is S}
\end{array}$$
(10)

can be considered as a primary indirect mood of the fourth figure for which one proposes as name *Camente*. By these two primary indirect syllogistic moods, the Aristotelian expression "in all the figures" also includes the fourth figure. A similar discussion can be made about the moods *Bramantip* and *Dimaris;* each of them results from a primary indirect mood whose names can be *Bramana*, with the same premisses like *Bramantip* and the conclusion "All P is S", respectively *Dimari*, with the same premisses like *Dimaris* and the conclusion "Some P is S".

# 3. Proposing new indirect syllogistic moods

Conversion and subalternation of the conclusion of certain syllogisms, as methods for getting other syllogisms, are specified by Aristotle himself in the text:

Because some syllogisms are universal and others particular, all universal syllogisms give more than one conclusion, and of the particular ones, the affirmative ones give more than a conclusion, while the negative ones give only the usual conclusion. (*An. pr.*, II, 1, 53a, 3-8).

According to the text, the universal syllogisms give two further conclusions, one by the conversion of the "usual conclusion" and the other by the subalternation of the "usual conclusion". The particular-affirmative syllogisms give only one conclusion obtained by conversion of "usual conclusion". By "usual conclusion" Aristotle means the conclusion that states the minor term depending on the major term, even if it is obtained by conversion of the conclusion resulting from the operations applied to the given premisses, as is the case of the mood *Camestres*.

The text just quoted is applied to all direct and indirect syllogisms. In this paper, will be discussed only the getting of the indirect syllogisms by conversion and subalternation. In the first figure, the indirect moods *Baralipton, Celantes* and *Dabitis* were obtained by conversion of the conclusions of the direct moods *Barbara, Celarent* and *Darii*, respectively (Didilescu and Botezatu 1976, 101). The conclusions of the three indirect moods are: "Some P is S", "No P is S" and "Some P is S", respectively. These moods were established by Theophrastus (Dumitriu 1969, 186).

If one follows the example of the direct moods of subalternation of their conclusion, a method by which a total of five direct subaltern moods have been obtained, other indirect syllogistic moods can be obtained in all the figures. For the first figure, as indirect subaltern mood can be considered *Celanto*, from *Celantes*, with the conclusion "Some P is not S".

In the second figure, can be obtained by conversion the indirect mood *Cesares* with the conclusion "No P is S" resulting from the conclusion of the direct mood *Cesare*. By subalternation of the conclusion

of the indirect moods *Cesares* and *Cameste*, will be getting other two indirect moods with the same conclusion "Some P is not S" and for which we propose the names *Cesareso* (to distinguish it from the direct subaltern mood *Cesaro*), respectively *Camesto*.

In the third figure two indirect moods can be obtained, both with the particular-affirmative conclusion "Some P is S" resulting by conversion of the particular-affirmative conclusion "Some S is P" of the direct moods *Darapti* and *Datisi*. As names are proposed *Daraptis* and *Datisis*. Indirect moods by subalternation cannot be obtained in this figure.

In the fourth figure, indirect syllogistic moods cannot be obtained by conversion, because three of the direct moods, as it was shown above, were obtained by conversion of primary indirect syllogisms. Indirect syllogistic moods by subalternation result from the proposed primary indirect moods *Bramana* and *Camente*. For these indirect moods, the names proposed are *Bramanip* and *Camento*; the first one has the conclusion "Some P is S" and the second one has the conclusion "Some P is not S".

# 4. Conclusions

These which are presented above can be summarized as follows:

- I. Primary indirect syllogistic moods are 13 of which:
  - a) *new-established*, 3 one in the second figure (*Brocamo*), one in the third figure (*Bramoco*) and one in the fourth figure (*Fimeno*);
  - b) recognized, 5 two in the first figure (*Fapesmo* and *Frisesomorum*), one in the second figure (*Firesmo*) and two in the third figure (*Fapemo* and *Frisemo*);
  - c) *proposed*, 5 one in the second figure (*Cameste*), one in the third figure (*Disami*) and three in the fourth figure (*Bramana*, *Camente*, *Dimari*).
- II. Indirect syllogistic moods by conversion are 6 of which:
  - a) recognized, 3 in the first figure (Baralipton, Celantes, Dabitis);
  - b) *proposed*, 3 one in the second figure (*Cesares*) and two in the third figure (*Daraptis*, *Datisis*).

III. Indirect syllogistic moods by subalternation are 5, all *proposed*: one in the first figure (*Celanto*), two in the second figure (*Cesareso, Camesto*) and two in the fourth figure (*Bramanip, Camento*).

Consequently, there are 24 possible indirect syllogistic moods of which only 8 are recognized. By admitting the 3 new-established indirect moods and the 13 proposed ones, each syllogistic figure will have 6 indirect moods, what shows a numerical equivalence between the direct and indirect moods.

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