RESISTING THE PARADOX OF INCREASE

CONSTANTIN PRICĂ[1]

Abstract: The fact that an object gains a new part, when before it didn't have it, is paradoxical. I argue that a holistic approach that considers change, defined as both increase and decrease of parts, can resist this unwanted implication. The approach requires understanding objects within the context of a world, be it finite or infinite. In such a world, objects can remain themselves even after exchanging parts. The primitive notion of essence (following in the tradition of Kit Fine) is central to this outcome. I argue that the actual world is similar to worlds that permit change, despite some concerns regarding vagueness.
Keywords: The Paradox of Increase, The Growing Argument, material constitution, essence

Introduction

The aim of this paper is to show one way of resisting the paradox of increase. Some metaphysical commitments are required, but I consider them by and large uncontroversial, especially because they can be developed in accord with physical discoveries and theories. Other ways to resist or even solve the paradox of increase have been proposed[2]. I will not go into them; the direction of my approach is quite different. Specifically, my approach is holistic and considers increase, decrease and more generally change only inside a finite or infinite world of

[1] PhD candidate, University of Bucharest. Contact: const.prica@gmail.com
[2] Olson (2006) is a comprehensive study of the controversial solutions (as considered by the author) to the paradox.

objects. This is in contrast with a restricted formulation of the paradox in terms of a number of objects[3] and no context.

I believe that, just as motion is possible even if Zeno's paradoxes suggest otherwise, change is possible and real in the actual world even if the paradoxes of increase and decrease would try to convince us otherwise. Trees, cats, humans, etc. do undergo change and are themselves throughout their lifespan. Real change is contrasted to apparent change, existing only at the level of language. I find it preferable to account for change at the metaphysical level of existence, over a piecemeal language analysis of propositions that contain change-like verbs[4].

I will start with the restricted formulation of the paradox, and then go on to expand it in two ways. First, by introducing a background (call it "world") to the objects, and second by introducing the opposite action of decrease among the objects. I will then try to show that objects inside a world, be it finite or infinite, can change (defined as the successive increases and decreases of parts). The notion of essence, understood as primitive[5], has an important role to play in how objects can be identical even if they change. At the end, I will tackle vagueness, and why it isn't metaphysically worrisome outside of some special cases.

1. The Paradox of Increase and the Paradox of Decrease

The Paradox of Increase or the Growing Argument, the name under which it is known from Antiquity[6], simply states that adding a new part to an object is impossible. A straightforward way of exemplifying this can be found in Chisholm (1979) and it involves an object A to which we attach B.

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[3] Olson (2006, 402-403) presents the puzzle as a *reductio ad absurdum* argument in six steps, starting with only three objects: A, B and C (identical with A at the start).

[4] Olson (2006), chapter 4, discusses how language can be made to account for change when real change is impossible.

[5] I will follow Fine's notion of essence as presented in "Essence and Modality" (1994a) and "Senses of Essence" (1994b).

[6] Rea (1995), note 8, presents the most likely origins of the Growing Argument.

> At the earlier time, t_1, A and B were separated; at the later time, t_2, they are conjoined. But what object became bigger? It was neither A nor B, for these things remained the same size they were before. And it was not AB for AB did not exist until A was joined with B. That is to say, AB did not have two different sizes, a smaller one at one time and a larger one at another. (Chisholm 1979, 158)

The assumption to note is that A and B stand for any objects or things[7] whatsoever. Let A stand for a heap of sand, upon which I add more sand. Does the original heap grow or a new heap with more sand in it replaces it on the table? Let A stand for a model ship without a figurehead, and B for the figurehead. Does attaching the figurehead B to the ship A make AB come into existence, or A, the ship, "already had B as a part then and we merely changed it from a disconnected or "scattered" object (like an archipelago) to a connected one" (Olson 2006, 391)?

A person being identical with its body is less controversial than the identity of the body with the sum or collection of molecules from which it is made of. Because with the next logical step, very reasonable, that a collection of molecules/particles is different after removing/adding or replacing molecules, we arrive at the conclusion that the person disappears after said (minor) change. Outside a deflationary view of reality, that has only the basic particles existing, such a general conclusion rings false.

Moral responsibility, in a certain sense, becomes a joking matter. One of the first formulations of the puzzle of increase is, according to (Rea 1995, 529), that of the comic playwright Epicharmus, and it involves a person wanting to collect a debt. The debtor argues that he isn't the same person as the one who contracted the sum of money and therefore shouldn't pay the sum back[8].

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[7] One way of attacking the argument is that B is not an object, but simply a detached part, that doesn't have existence outside of A. Peter van Inwagen in "The Doctrine of Arbitrary Undetached Parts" (1981) discusses this approach.

[8] The debtor isn't the same because his atoms or molecules have changed. Thanks to a reviewer for pointing out that there are other ways of explaining why the debtor isn't the same, such as different time slices that compose him.

Removing a part of an object is deemed impossible as well. This is the paradox of decrease[9]. To exemplify it, take object A that has two parts B and C. After removing B, does A still exist? It seems to be identical now with C, even though before removing B, C didn't have clear persistence conditions or an identity. Do note that what was previously a connected object (A at the start) is now simply a scattered object, by having its two parts simply at a distance from one another. To emphasize further the absurdity of impossible decrease: the action of plucking a hair from your head annihilates or scatters you.

The two puzzles express the impossibility of change. Olson calls this "the doctrine of mereological constancy" and defines it as: "Necessarily, if x is a part of y at some time, then x is a part of y at every time when y exists" (Olson 2006, 392). Object cannot replace their parts, because they would need first to remove a part, and second to add a new one.

Mereological constancy or essentialism[10] goes against common intuitions and beliefs regarding human beings and inanimate objects. I've grown over a period of 32 years, even if all of the atoms that compose me have been replaced during this time. I have had the same parents even after they've had numerous haircuts over the years. I still use the same laptop even after replacing its hard disk drive. I can go on with similar examples.

The picture seems grim, but I will try to show that it isn't. The first move is to specify a context for change.

2. Finite and Infinite Worlds

Let's consider a collection of objects and call it a world. All objects are made of parts, and any parts can be connected or scattered[11] between the

[9] Some other common names for it are: The Amputation Paradox (Olson 2006, 392), The Dion/Theon Puzzle first formulated by Chrysippus according to Sedley (1982), The Tibbles/Tib puzzle first published in Wiggins (1968).

[10] Olson argues for a difference between the two in Olson (2006, 392-393). I don't consider this to have great impact on my approach.

[11] So as to avoid the usage of add or remove.

objects. Parts can be infinitely or not disconnected. If they are an infinite number of objects (or parts) the world is infinite, otherwise it is finite[12].

The first important assumption, that I consider reasonable, is to consider both adding and removing of parts together. In an important sense, they can cancel out[13].

If the two actions are equally likely to happen in a finite world, then that world is static. Therefore, change is a localized property of the world, but globally the two puzzles are satisfied. If they are not as likely to occur, change remains localized, but the world either falls apart or unifies into a single object.

For an infinite world, change is also at least a local feature. The type of infinity the world exhibits[14] is a new dimension to be considered alongside how likely objects are to add or subtract parts. But, all in all, the same outcomes as for a finite world can be described for an infinite world as well[15].

Considering increase and decrease together, within a collection of objects, is just the first step towards an account of change. Identity criteria are required for objects, so that they survive increase and decrease in the world (be it finite or infinite).

3. Criteria for Object Identity

Consider a finite world with only two objects, made up of only two parts: A_1A_2 (A) and B_1B_2 (B). By switching the parts around, the possible combinations, not considering a unique order, are: A_1A_2, B_1B_2, A_1B_1, A_1B_2,

[12] Think of the elementary particles (i.e. electrons, neutrinos, etc.) as the building blocks of a world not infinitely divisible.

[13] A helpful analogy would be a game of Lego, where the player is trying to construct an object by adding and removing pieces. Because the two actions have opposite effects, the player cannot simply add some piece and then remove it afterwards. He wouldn't advance in any real way. He needs some succession of both adding and removing of pieces to form an object.

[14] I will assume an infinite world to have only a countable infinite number of objects and be made of elemental particles that aren't divisible further.

[15] The infinite world has its parts or objects from the "beginning", there is no way to infinitely generate parts or objects from nothing.

A_2B_1, A_2B_2. One example of grouping of replacements that has both A and B survive change is: A (A_1A_2, A_1B_1, A_1B_2), and B (B_1B_2, A_2B_1, A_2B_2). Any of the other groupings also work as criteria of object identity.

In finite worlds, defining identity as an exhaustive list of possible parts replacements is easy. In a sense, objects are no more than these replacements. In an infinite world, the notion of essence, understood as primitive[16], can play a similar role. Some replacements can make an object not be itself (lose its identity), while other replacements will have the object essentially be the same at the end. Determining which replacements are part of the essence of an object is not metaphysically relevant now[17]. What is important is that essence draws a sinuous line in the sand between some that are identity-preserving and some that are not. In the case of abstract objects, such as sets, the demarcation is quite clear. The singleton S cannot suffer increase or decrease and still be itself[18]. In the case of human beings, their hair color or hair length are decisively not essential. Because the body is made of atoms, losing some atoms, or replacing them with other atoms, is possible in some cases[19]. Living things are fuzzier than inanimate objects in regard to their essential parts.

Important to note that essence as a criterion for identity across change is informative only in the context of world. When considering an object outside of any context (the original formulations of the paradoxes), its essence is at most its haecceity. And therefore, it is just a way to ignore the problem of change. Growth and change in a one-object world are nonsensical notions.

The picture sketched so far, of how objects suffer change, still needs some work. First, if essence is only internal to the objects themselves, change can be superfluous. For example, consider a world made up of self-sufficient objects. Second, change can seem *ad hoc*, even if present. This is most apparent in a finite world, as we can exhaustively enumerate all the possible configurations between object parts and

16 In the tradition of Fine (1994a, 1994b), Correia (2006), Lowe (2008).

17 It is of course epistemologically desirable.

18 I consider all sets to be essentially incapable of change, more exactly of adding and removing of parts.

19 The atoms that form the head are intuitively more important than others in the body.

allocate them (*ad hoc*) to one object or another. In an infinite world, determining which configuration of parts belongs to which objects can be quite simple as well[20].

Both observations, that essence isn't sufficient or necessary for change inside a world, are hard to dismiss. A static world, where change isn't present, and a world without objects, just configurations of parts, are both in agreement to the paradoxes of change. The paradoxes shouldn't be dismissed, just resisted as they don't have any power inside other types of worlds.

4. Fleshing out Essence

Can any object share parts with any other object? In the actual world, trees, tables and humans don't exchange their atoms with trees, tables, or humans on the other side of Earth, or rocks on the Moon, or underwater ice lakes on Mars. Essential for objects, in general, is to not get scattered[21], to have a certain spatial and temporal continuity. This continuity gives objects a certain lifetime.

The frequency of exchange of parts characterizes one aspect of an object's lifetime, that of temporal continuity. A higher degree of exchange of parts translates into greater uncertainty of object configurations. A low degree of exchange makes objects rigid. Of course, the exchange of parts need not be investigated at the scale of the world and can be localized to regions of the world (certain sub-collections of objects).

The other aspect, the spatial uniformity, is characterized by the average distance between any two objects that exchange parts. A

[20] Consider the natural numbers, and distinguishing between odd numbers and even numbers.

[21] Collections of coins or stamps are examples of objects that can get scattered. By adding a new coin to the collection, does the collection grow or get replaced with a collection that has more coins? I think the former option is true. After all, essential to the collection is not what is essential to the set of coins that constitute it. A collection of ancient roman coins can be defined as the result of the activity of gathering together, with the purpose of preserving, all the roman coins created. The set of roman coins, like any other set, has its members essentially, and therefore cannot survive growth or decrease.

scattered world has objects from afar exchange parts. The highest value means any object can exchange parts with any other object. Similar with time continuity, spatial uniformity need not be investigated only at the scale of the entire world.

Spatial uniformity and temporal continuity are criteria for identity when they are part of the essence of an object. Arguably, a living thing, be it human, or any other animal, plant, insect etc. exists for a time continuously and, more or less, in a certain body. In other words, it is what it is for a certain time and in a certain spatially extended body[22]. And they essentially can't survive being unmade and put back together. Tables or statues also cannot survive annihilating changes, but in contrast to living things, can be dismembered and put back together.

Immaterial objects, such as the number 2 or prime number theory, are unchangeable because they don't have parts. Of course, in set theory, 2 can be defined based on the set that defines the number 1, and also the theory of prime numbers refers to other objects, specifically the prime numbers. But these are actually constituents of the real definitions of those objects. According to Fine, the constituents of the essence of an object are ontologically depended upon for the object in question to exist (Fine 1995, 275-276). In this sense, I consider intuitions regarding parts of immaterial objects to be more aptly intuitions about the "parents" of immaterial objects.

The actual world can be characterized as a non-rigid world, with one caveat. The distinction between essential and accidental properties is not necessarily a feature of objects in our world[23]. Material objects cease to exist, or at least cease to be causally relevant, if reduced to elementary particles that don't change, that only interact among each other. But in common language we heavily employ terms that refer to material objects, which we don't translate immediately into terms about

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[22] An endurantist perspective is assumed. I will not go into a perdurantist account of objects or worlds. Lewis (1986) discusses a perdurantist account of objects.

[23] And even if it is, the distinction isn't necessarily primitive. I see no reason to exclude other notions of essence from the analysis of a world with change. However, I suspect, that in the end, some other form of brute facts about essences must be relied upon to make an object be itself after change.

elemental particles[24]. This fact points to objects such as trees, cars or stars as actually existing in the world.

That A is a part of AB at one time and part of another object AC at a different time makes sense only if there is a "border" to separate the two objects. If such a border cannot clearly be discerned, then how can an object increase in parts and another decrease in parts?

5. Vague objects

Let's consider again the finite world with only two objects made up each of two parts: A_1A_2 (A) and B_1B_2 (B). Vagueness at the metaphysical level can be exemplified in this world by having both objects share at least one configuration: A (A_1A_2, A_1B_1, A_1B_2, $\mathbf{A_2B_1}$), and B (B_1B_2, $\mathbf{A_2B_1}$, A_2B_2). In the overlapping configuration (i.e. A_2B_1), A seems to have to doubled, since A_1B_2 (the remaining parts only configuration) is also A. Object B has disappeared, only to possibly reappear after a change to object A (the moving of parts from A to A).

Consider another case, that in which the two objects share all the mismatched configurations (A_1A_2, B_1B_2, A_1B_1, A_1B_2, A_2B_1, A_2B_2). Only when A is A_1A_2, and B is B_1B_2 the two objects are different. And because of the finite nature of the world, they are also synchronized in their identity. Necessarily when A is itself, B is itself.

The problem of vagueness in an infinite world isn't solved by appealing to essences in at least two cases. First, consider objects with not clearly defined "borders", like clouds[25] or puffs of smoke. The exact demarcation between water droplets inside the cloud and outside the cloud is not essential. Neither is defining a clear line between my right arm and the torso (or the rest of my body). For clouds and bodies, an approximation of their parts is sufficient most of the time.

Second, consider the case of simultaneous definitions: Sherlock Holmes and Doctor Watson. One object is essentially dependent on the

[24] It is debatable if such a translation is even possible. Where would we find a native speaker of elemental particles language to converse with and learn from?

[25] David Lewis discusses objects with vague borders in Lewis (1993).

other, and vice versa[26]. Similar with the finite world example, Sherlock Holmes cannot be identical with itself if Watson is not identical with itself. The two characters should be considered together in regard to their identity.

In both types of situations, vagueness can be better tackled at the level of language. Propositions about the exact border of a cloud seem meaningless. So do questions regarding the way in which Sherlock Holmes and Watson change. At the metaphysical level, objects would then be clearly defined (made of some parts as opposed to others), and language would be an imperfect way to express their identity after change.

Conclusion

The puzzles of change, when analyzed in the context of a collection of objects, a world, can be better understood and their implications can be resisted. I have sketched a holistic approach for increase and decrease considered together in a finite or infinite world.

To make objects be more than *ad hoc* configurations of parts, I rely on the notion of essence understood as primitive. The intuition is that some changes in parts lead to an object no longer being itself, e.g. removing sand for a heap of sand. While, after other changes in parts, the object remains the same, e.g. plucking a hair from the head of person. Objects have a certain lifetime in the world, characterized, in most cases, by space-time continuity.

The difference between changes that destroy the object and changes that do not is not always clear. I have gone over possible causes of vagueness at the metaphysical level, i.e. objects with no clearly defined borders and objects defined simultaneously.

I have also touched upon the similarity between the actual world and worlds (either finite or infinite) that permit change. Everyday beliefs and common language object-terms are a strong incentive to go further,

[26] Fine (1995, 282-283) discusses simultaneous definitions; objects are defined together, so as to not generate a cycle of ontological dependence.

and identify the actual world with one of the types of worlds that resists the paradoxes of change.

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