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Universitatea din București, Facultatea de Filosofie
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E-mail: redactia@srfa.ro

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THREE LESSONS ABOUT MODEL THEORY

SILVIU VELICA[1]

Abstract: In this paper, I will point out some consequences of the recent discussions in the philosophy of model theory concerning the possible stances one might take toward the philosophical significance of model theoretical results. The main lesson of these discussions seems to be that model theoretical results are significant philosophically only if we can justify their use independently of model theory. This follows from the fact that both moderate modelism and model theoretical scepticism are untenable. I will be using the problem of the categoricity of arithmetic throughout the paper as a case study.
Keywords: *Philosophy of Model Theory, Categoricity, Modelism, Model Theoretical Scepticism*

1. Introduction

Over the past few decades, model theory has known a steady increase in both formal results and philosophical interest[2]. One of the main points of attraction for philosophers has been the problem of categoricity for various mathematical theories, and especially for arithmetic, since, if mathematical practice and informal discourse seem to suggest that we are dealing with a specific mathematical structure, then, the argument goes, we ought to be able to formally pin down that structure using model theoretical notions[3]. This intuitive line of thought quickly stumbles into

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[1] PhD candidate, University of Bucharest. This paper is part of my PhD research on categoricity and IF logic. Contact: velicasilviu@gmail.com

[2] I will not delve into the historical development of model theory or the notion of categoricity. Excellent historical accounts can be found in Awodey and Reck 2002 and Hodges 2018.

[3] Some alternative model theoretical notions have been proposed, but categoricity has the most prominent history. On the reasons for interpreting informal structure talk as

difficulties brought about through model theory itself: for one thing, model theory shows decisively that first-order logic simply cannot provide categorical characterizations of structures with infinite domains, since it satisfies the Compactness and Löwenheim-Skolem theorems. Then, if we want to go beyond first-order logic, we need stronger assumptions in order to specify those stronger logics and their semantics. At this point, for reasons to be explained below, it becomes clear that we cannot hope to succeed in our attempts to connect formal and informal structure talk with the tools provided by model theory alone.

The main purpose of this paper is to stress the implications of the impossibility mentioned at the end of the previous paragraph and to offer a clear picture of what must be done if we are to pursue the intuitive argument given above. The philosophical discussion up to that point is greatly indebted to the detailed analysis given in Part B ("Categoricity") of Tim Button and Sean Walsh's 2018 book, *Philosophy and Model Theory*.

The following sections will deal with the motivation behind the interest in categoricity results (2) and the formal constraints which constitute the limits on determinacy of reference from a model theoretical perspective (3). The next two sections are concerned with philosophical attitudes toward model theory, particularly what will be called *moderate modelism* (4) and model theoretical scepticism (5). The lessons which, in my opinion, can be drawn from all this are given in section 6.

2. Three roles for categoricity

Without delving into the technical background (the required definitions are available in most mathematical logic textbooks), we can say that categoricity is a property of an axiomatic system by which all the structures (or models) which satisfy the axioms are isomorphic between them, i.e. the theory in question determines a unique structure up to isomorphism. It is of course important to understand why categoricity is

isomorphism types (which is what a categoricity result helps secure), see section 3 (the same section also mentions two alternative notions). Also, the focus on arithmetic in what follows should be taken, at least for now, as a simple case study.

desirable. Toby Meadows mentions three possible reasons for this (see Meadows 2013, 526-527), which can be summarized as follows:

First, we may want to show that there is a unique structure corresponding to some mathematical intuition or practice (Meadows 2013, 526). What I make of Meadows' explanations on this point is this: Suppose we are inclined to believe that, when we are doing arithmetic, for instance, we are in fact talking about a *particular structure*. Having a categorical theory determining that structure would count as an argument for its existence in the following sense: if I can determinately refer to a structure, that structure exists. It's not a particularly compelling argument, but it does make our pre-theoretical belief more plausible, which in turn is enough to make categoricity desirable.

Second, and in an opposite direction to the first reason, we may already believe that there is a structure to which some practice or intuition corresponds and we would be interested in a theory capable of referring to this structure (Meadows 2013, 526). In other words, the argument would go like this: if this structure exists, I should be able to refer to it determinately. Note that this line of argument is concerned more with the choice of logic than with the structure as such. So we might say that a categorical theory whose unique model is our initial structure makes that theory and the logic it is formulated in *appropriate* for our investigations concerning that structure. Of course, one can be committed to a logic which can't deliver categoricity, but then we would have to find other ways of determining our structure.[4]

Finally, categoricity can be used as a criterion for classifying theories, separating those which we expect to have different, non-isomorphic models (group theory, for instance) from those which we intend to determine a unique structure (arithmetic is usually seen like this) (Meadows 2013, 527) – in Shapiro's terms, separating algebraic from non-algebraic theories (see Shapiro 1997, 40-41). This use of categoricity presupposes that we have already made our choice of logic and also that this logic is strong enough that we don't end up saying that all theories with infinite domain are algebraic in Shapiro's sense, i.e. we expect them to have non-isomorphic models, as we would in first-order logic.

[4] Some possible attitudes towards the relationship between the choice of logic and categoricity are considered in Button and Walsh (2016, 12-13).

None of the reasons just mentioned makes categoricity an absolutely indispensable property for our theories – at least, not without further presuppositions about model theory and mathematics in general (we will get to these further on). But they do point to some of the problems a satisfactory account of our mathematical practice should address. In short, we are dealing with an intriguing issue concerning the relationship between our mathematical practice or intuitions, arithmetic, and our choice of logic: if we do have such a good pre-theoretical understanding of the natural number structure, then should it not be easy to uniquely determine this structure in our formal treatment of arithmetic?

3. Why stop at isomorphism?

Some additional clarifications are in order before moving on to the overarching philosophical problems. One of them concerns our inability to achieve any form of referential determinacy which goes beyond an isomorphism type[5]: it is one of the most famous results in the philosophy of mathematics, due to Benacerraf, that it is completely irrelevant from the perspective of mathematical practice what sort of objects the natural numbers are, as long as certain structural constraints are met, and, therefore, that "any recursive sequence whatever would do" (Benacerraf 1965, 69). This result is important in the current context because it is grounded on model theoretical notions, namely the so-called Push-Through Construction (PTC):

The Push-Through Construction. (Button and Walsh 2016, 2) Let $\mathcal{L}$ be any signature, let $\mathcal{M}$ be any $\mathcal{L}$-structure with underlying domain M, and let $\pi: M \to N$ be any bijection. One can use π to induce *another* $\mathcal{L}$-structure $\mathcal{N}$ with underlying domain N, just by stipulating that $s^{\mathcal{N}} = \pi(s^{\mathcal{M}})$ for each $\mathcal{L}$-symbol s. Having done this, one can check that $\pi: \mathcal{M} \to \mathcal{N}$ is an isomorphism.

[5] Some philosophers might still insist that numbers are some definite entities, but they would have to identify them by means independent of model theory; the inability I mentioned here is from a model theoretical perspective.

In other words, what the PTC offers is a straightforward method for creating a new structure from a given one, with the copy being isomorphic to the original. Substituting one structure for another, as long as they are isomorphic, is of no consequence whatever from the point of view of mathematical practice. This strongly suggests that mathematical practice cannot determine the reference of mathematical discourse beyond an isomorphism type, which in turn seems to motivate the idea that "mathematical structures, as discussed informally by mathematicians, are best explicated by isomorphism types" (Button and Walsh 2016, 3).

There is another famous use of the Push-Through Construction employed by Putnam (the so-called Permutation Argument) by which he argues that there is nothing in model theory which fixes the references of our terms. It is important to explain this argument here, since it will play a central role in the following philosophical discussion. The argument goes like this (I will use the toy model given in Button and Walsh 2018, 39): suppose we have a theory T consisting of three sentences $\{Ca, Cb, \sim Cc\}$. Now take the model $\mathcal{M}$ to be an interpretation of our theory T in the following way: $\mathcal{M}$ assigns to a the object α, to b the object β, and to c the object γ; and, of course, the extension of C in $\mathcal{M}$ is $\{\alpha, \beta\}$. Since $\mathcal{M}$ makes all the sentences in T true, we might be inclined to use this model to explain the reference of each term in T. However, using PTC, we can construct another model $\mathcal{N}$ starting from $\mathcal{M}$, which also makes our theory true, but which disagrees on the reference of each term: if $\mathcal{N}$ assigns to a the object β, to b the object γ, and to c the object α, then it can simply assign to C the extension $\{\beta, \gamma\}$. So Ca is true in $\mathcal{M}$ because a 'refers' to α and α is in C's extension; it is also true in $\mathcal{N}$ because a 'refers' to β and β is in C's extension; the same goes for Cb and $\sim Cc$.

If this is all the information we are given, then we simply can't tell whether $\mathcal{M}$ or $\mathcal{N}$ is the intended interpretation for T, which means that we also can't tell which model represents the "correct" reference relation. This shows quite clearly that the notion of *truth in a model* is not sufficient to deliver determinacy of reference (Putnam 1977, 485-486). This does not mean that we can *never* have determinate reference – we might invoke notions such as causality which would allow us to identify

the correct interpretation[6]. But note, and this is the crucial point, that there is nothing we can do by model theory *alone* to choose between the two models. So if we want to say that we reach the natural number structure through model theory, then we simply cannot go beyond the isomorphism type of the standard model, unless we invoke some notion external to model theory – perhaps a certain causal relation with the numbers themselves (but this carries so much metaphysical baggage that few philosophers would be willing to pursue this line of thought).

Before we continue, it is worth mentioning that there are other model theoretical notions which have been proposed as better candidates to represent our pre-theoretical notion of structure and which are not so restrictive as isomorphism. One such notion is *definitional equivalence*, which is satisfied by two structures if they have the same domain and are definable in one another (Button and Walsh 2018, 108). But definitional equivalence is problematic as a representation of structure, since it depends upon the choice of logic, the structures must share the same domain, and it also doesn't allow us to build new structures by taking equivalence classes (Button and Walsh 2018, 109-110). A different notion we could use is *interpretability*: a structure $\mathcal{M}$ is interpretable in a structure $\mathcal{N}$ iff it is isomorphic to a structure $\mathcal{N}'$ formed by taking definable equivalence classes over the domain of $\mathcal{N}$ (Button and Walsh 2018, 111). Again, however, interpretability depends on facts about the ambient metatheory – it might turn out that the natural numbers and the integers are seen as identical structures, since one can be interpreted in the other, although we might not agree to this in informal discussion (Button and Walsh 2018, 113-114). In what follows, we will only be interested in isomorphism types, since the problems we will be dealing with concern model theory in general.

[6] There is a wider discussion here about whether such criteria actually work, but we are currently focusing strictly on model theory (see Button and Walsh 2018, 40-41).

4. Modelism[7]

I will borrow the terminology introduced by Button and Walsh in their 2018 *Philosophy and Model Theory* and use the term 'modelism' to refer to the idea that the mathematicians' informal structure-talk is to be understood in some model theoretical terms like isomorphism, definability, interpretability or some such notion (Button and Walsh 2018, 139). Since the problems with modelism concern the entirety of model theory, it will not matter which notion we take to explain informal structure-talk, so we will focus on isomorphism. Note that explaining structure-talk through isomorphism types makes formal results like categoricity even more valuable than what was discussed before – that is, if we still want to claim that it makes sense to talk about, say, the natural number structure.

Given the modelist thesis, the question immediately arises as to how we can pick out a particular structure, i.e. a particular isomorphism type. This is, again in the terms of Button and Walsh, a doxological problem: we are not concerned with knowledge, but with belief (Button and Walsh 2018, 145-146), and we require "an account of how we are able to refer to any of the entities which populate [the modelist's] mathematical ontology" (Button and Walsh 2018, 146).[8]

Modelism is not particularly problematic in general, but it becomes very much so if we add the further requirement that the modelist thesis is to be carried out using exclusively model theoretical notions – this is called *moderate modelism* (Button and Walsh 2018, 42, 139). In broad outline, the problem is this: if we want to pin down the natural number structure as moderate modelists, we cannot use first-order logic, since we lack categoricity; so the natural approach seems to be to use a stronger logic – either second-order or else some intermediate logic which can deliver categoricity. However, these

[7] The current section and the next draw heavily from the detailed discussion in Button and Walsh (2018), chapters 6, 7, and 9. I believe that no discussion of categoricity is complete without addressing the philosophical concerns Button and Walsh raise.

[8] Button and Walsh actually consider two versions of modelism, but the general problem is the same for both versions, and the distinction is irrelevant to my purposes, so I will simply ignore it. For details, see Button and Walsh (2018, 144-150).

stronger logics need to be somehow specified, and this is an operation which essentially involves mathematical notions – notions which, by the moderate modelist's light, must be characterized model theoretically. In short, moderate modelism must somehow be in possession of the theoretical notions needed to specify those very notions. This is worth a closer investigation.

Suppose the moderate modelist wants to use full second-order logic and appeal to Dedekind's categoricity theorem to pin down the standard model of arithmetic. In order to do this, she must first specify the semantics of full second-order logic and, in particular, make sure that she is not unknowingly using Henkin semantics (Button and Walsh 2018, 159). Henkin semantics is a more restricted version of the full semantics in that quantification over second-order variables does not range over all possible subsets of the domain, but over a given set of such subsets (Button and Walsh 2018, 24). This seemingly small restriction has important consequences, however, since the Löwenheim-Skolem theorem holds for Henkin semantics, which also implies that Dedekind's categoricity theorem fails (Button and Walsh 2018, 158) – so it is imperative for the moderate modelist to distinguish between the two. Now, since our hypothetical philosopher is a *moderate*, she cannot invoke any kind of mathematical intuition, so she must use model theory alone to do this (Button and Walsh 2018, 152): she needs a strong enough metatheory, which should, in turn, be at least second-order – but then the problem just reiterates itself; or, in Putnam's words, "the 'intended' interpretation of the second-order formalism is not fixed by the use of the formalism" (Putnam 1980, 481). Again, there might be ways to ensure that we are using the full semantics (Putnam proposes such a solution for natural language – Putnam 1980, 482), but they are not open to the moderate modelist. The moderate modelist *must* answer the doxological challenge "by description", and this description must be given in model theoretical terms (Button and Walsh 2018, 152).

To make the point even more forcefully, recall from the previous section the way Putnam used the Push-Through Construction to argue that model theory alone cannot provide referential determinacy. The description which the moderate modelist might offer to distinguish full from Henkin semantics, given completely in model theoretical terms, is

just another theory to which we can apply the Permutation Argument – simply reinterpret it via the Push Through Construction (Button and Walsh 2018, 159). This is the well-known *just more theory* manoeuvre (Putnam 1980, 477), and the moderate modelist has no way of dealing with it. So, full second-order logic is out of reach for moderate modelism.

To close off this section, let us quickly take a look at some alternatives to second-order logic from a moderate modelist perspective. If we try to use a multi-sorted first-order logic, there are two ways to view the problem: first, if the metatheory is itself first-order, then it will admit non-standard interpretations by virtue of the Löwenheim-Skolem and Compactness theorems (Meadows 2013, 530); otherwise, if the metatheory is second-order, we have the same problems as above. It is easy to see why the problem shifts to the semantics of the metatheory, since one of the two sorts needed to secure categoricity effectively replaces universal second-order quantification: instead of quantifying, we simply add the range of the second-order quantifier to the domain as a second sort of object. But to specify this, we need *full* second-order quantification in the metatheory.

Similar observations are applicable to other approaches: if we want to allow a one-place second-order variable, we need to specify its semantics, which should be the full rather than Henkin semantics (Button and Walsh 2018, 162-163); if we want to impose restrictions on second-order quantifiers (make them range only over finite subsets) or employ generalized quantifiers saying that there are finitely many φs or Härtig's quantifier (*there are exactly as many* φ*s as there are* ψ*s*), then we need to be able to specify cardinality in general (Button and Walsh 2018, 163); the same goes for allowing infinite conjunctions and disjunctions. Tennenbaum's theorem, linking the computability of addition and multiplication to the standard model of arithmetic, is likewise inaccessible, since defining computability already presupposes "precisely the arithmetical notions it was supposed to vouchsafe" (Button and Walsh 2018, 164)[9]. Finally, taking the successor function and the *less than* operator as logical operators fixes nothing, since we still

[9] A more comprehensive critique of the use of Tennenbaum's theorem in this context can be found in Dean (2013), especially pages 10-11 and 19-20.

have to specify the semantics of our language. Most of these approaches can be described, with Read, as "[shifting] the problem from the identification of postulates characterizing [the natural number structure] categorically ('completely') into the semantics and model theory of the logic used to state the postulates" (Read 1997, 91).

5. Model theoretical scepticism

The discussion in the previous section leads us to conclude that

> [moderate modelists] need to explain how we grasp certain mathematical concepts. They must answer by invoking some categoricity theorem. But to prove categoricity, they must spell out the semantics of their chosen logic […]. In short: the moderate modelist's attempts to go beyond first-order logic invariably amount to *just more mathematical theory*. (Button and Walsh 2018, 164)

To be perfectly clear, the issue is not that model theory cannot provide categorical characterizations of the standard model of arithmetic – there are plenty of options here; the issue is that a *moderate modelist* has no way to access the theoretical notions needed in order to employ those model theoretical results. In other words, we need a way into model theory that is independent of model theory, otherwise we simply cannot use it.

At this point, if we take moderate modelism seriously, we might be inclined to accept a form of model theoretical scepticism and claim that we simply cannot pin down the natural number structure, since we don't have access to the theoretical notions we need for this purpose (Button and Walsh 2018, 203-205). It is important to keep in mind that this sceptical position is reached through explicit use of model theory: in the arguments against moderate modelism given above, we employed the distinction between the full and Henkin semantics for second-order logic, notions of cardinality and so on. This implies both that the moderate modelist cannot formulate this sceptical position, for the same reasons for which she cannot pin down the natural number structure, and that (at least this variety of) model theoretical scepticism is

incoherent[10]. Therefore, one simply cannot be a moderate modelist, in the sense described above: modelism requires that we use model theory, and moderation requires that we use *only* model theory. Taken together, these requirements leave us with no way to access the resources we need in order to meaningfully talk about arithmetic, regardless of whether we believe there is such a thing as the natural number structure or we want to deny it or even be sceptical about it.

6. Where we stand

It should be clear by now that moderate modelism is incoherent, but this leaves us with a choice between rejecting modelism or rejecting moderation (Button and Walsh 2018, 140). In what follows, I will point out some of the lessons that can be drawn from the preceding discussion from a modelist perspective. The lessons are relevant to our general goal of pinning down the natural number structure, so, with this in mind, I claim that we must accept the following conclusions:

1. Any logic is *formally* unobjectionable. What I mean by this is that the presence or absence of a formal result is never desirable or undesirable on its own account: we always need a non-formal justification for the desirability (or lack thereof) of the formal result in question[11]. Without such a justification, a logic can only offer empty formal derivations (see the comments in Sandu 2015, 103-104). This also means that no logic is preferable to any other *unless* we already know what 'preferable' means. This is what Button and Walsh refer to with the "rule of thumb" they mention: "it is impossible to extract philosophical

[10] One could conceive a more general sceptical position, an 'ineffable' scepticism, to which I offer no reply here. A much more elaborate discussion can be found in Button and Walsh (2018, 206-211). They offer two arguments against model theoretical scepticism, but the basic idea is the same, namely that this sort of scepticism presupposes resources which allow us "to brush aside the sceptical concerns" (Button and Walsh 2018, 208). See also the comments in Button and Smith (2012, 119-120).

[11] This can be taken to extremes – after all, it is a matter of our fundamental beliefs about the world that we are interested primarily in *consistent* logics, and this is true even for paraconsistent logics.

juice from a piece of pure mathematics, without invoking some philosophical thesis" (Button and Walsh 2018, 156). This leads to our second conclusion:

2. Philosophical theses cannot be justified by formal results *alone*. This is essentially the strategy of moderate modelism, and we have seen why it fails. This does not mean that formal results can never play a part in philosophical arguments – suppose we had strong reasons to believe that we cannot go beyond first-order logic; then we might use the existence of non-standard models for arithmetic as an argument that there is no such thing as *the* natural number structure. However, this does mean that we must always justify the use of formal results *before* we employ them, and this by means independent of those results (perhaps through some sort of mathematical intuition). It is philosophical theses that make formal results significant, not the other way around.[12]

3. If modelism is possible at all, then at least some model theoretical notions must be accessible to us from outside of model theory. In other words, we must fix the accessibility problem which moderate modelism faces before we use model theory to say anything about arithmetic. Renouncing moderation opens up our ability to invoke external justifications, such as intuitions, practices, maybe even causal relations etc. It is also mandatory to acknowledge that invoking external justifications is the only way in which we can maintain a modelist approach – otherwise we end up in the moderate modelist's predicament. Which particular notions are best suited in this position is obviously up for debate.

In short, a modelist may use any logic as long as she provides some external justification of our ability to employ that logic. She may then make further use of that logic in philosophical arguments, provided she avoids the sort of circularity that plagues moderate modelism.

[12] Here, I only address philosophical significance – the kind of significance that a philosophical argument which uses a formal result presupposes of that result. Of course, there are other ways in which formal results can be significant.

7. Concluding remarks

Let us now go over the main points of the preceding discussion. The basic idea is that model theory can be fruitfully used in philosophical discussions about arithmetic. When trying to articulate this idea into an actual philosophical argument, however, we encounter various difficulties, which are not unsolvable, but they do impose important constraints on what can actually be achieved philosophically by means of model theory. Some of these difficulties are purely technical – the expressive limitations of first-order logic (especially the lack of categoricity for structures with infinite domains) is probably the most significant example. But the formal difficulties are easily overcome if we are willing to go beyond first-order logic and use a more powerful logic. However, these alternatives are essentially more complex than first-order logic, and we need to make sure we have (in the broad sense of being accessible to the understanding) the theoretical resources required to specify them.

Without reiterating the discussion in sections 4 and 5, it is crucial to understand its main conclusion, namely that moderate modelism is impossible, and the important consequences it has. The impossibility of moderate modelism shows quite clearly that *if* we want to be modelists at all (i.e. to use model theoretical results in philosophical discussions), then we have to employ *some* philosophical principle which would allow us access to the model theoretical notions we need from outside of model theory. This also means that modelists need not worry themselves with objections coming from a moderate perspective, since the latter is simply incompatible with modelism, and, as such, modelists could never satisfy a moderate. But this still leaves enough room for objections from one modelist to another, since it will still be plausible to claim that some approaches assume too much compared to others. In any case, the upshot of all this is that modelism is still a reasonable attitude in the philosophy of mathematics, as long as we understand the constraints it is subject to.

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THE PLATONIC RECEPTACLE: BETWEEN PURE MEDIALITY AND DETERMINING CAUSE

OCTAVIAN PURIC[1]

Abstract: In this paper I will try to answer two different questions that stem from Plato's characterization of the Receptacle in the *Timaeus*. The first originates from the description of the Receptacle as a "characterless sort of thing" (50e5-6, 51a5) that "receives all things" (51b) without "showing its own face" (50e4). This lets one assume that whatever character the Receptacle will receive from the Forms, the resulting image will have that exact character and will in turn be an accurate depiction of its Form. Yet, this conflicts with the fact that Plato describes Forms and particulars in strikingly contrasting manners. Thus, the first question will be: What accounts for the differences in character between Form and particular in light of the Receptacles' pure receptivity? The second question asks as to the nature of the common character that binds the image to its Form: If the image and its model are differentiated by opposing characteristics, what is the common aspect in which an image resembles its model, making it of one particular model rather than of another? Before tackling these questions, I will first review three of the more influential interpretations of the nature of Forms and particulars in the following order: The Approximation View, the Unqualified Exemplar interpretation, and the Model-Image view. I will side with the latter, arguing that it provides the most satisfactory account of Plato's thought. Placing my approach in the theoretical framework of the Model-Image view I will argue that the first question can be solved in the following way. First, I will maintain that the Receptacle need not be characterless in all respects, but only in those in which it is to receive the characters of the Forms. Secondly, I propose that the image has two inherent types of determining sources, a) the formal determination that comes from its model, and b) the medial determination that stems from the medium to which it belongs. I will argue that it is due to the medial determination of the Receptacle that the image has all the contrary properties that oppose it to the Form. Regarding the second question, I will maintain that as it stands at a moment, the Model-Image theory cannot provide a satisfactory account.
Keywords: image-model, particular-form, receptacle, participation, approximation, unqualified, medial aspect, formal aspect.

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[1] PhD candidate, University of Bucharest. Contact: octaviandan91@gmail.com

1. Introduction

The Platonic Receptacle has received two main interpretations throughout the secondary literature. The first, following Aristotle, has construed it as a badly thought of account of matter. Diverging from this reading, later interprets[2] following Cornford (1997) took Plato to have in mind a type of medium in which the Forms are received as images, a function that is best exemplified by analogy with a mirror[3]. In this article I will follow the latter interpretation.

The Receptacle acts as a medium at least in so much as it counts as the necessary counterpart alongside the Form to the genesis of the image. It is that in which the images of the Forms, Plato's metaphor for the phenomenal world, come to be. Timaeus describes it as a "characterless sort of thing" (*Timaeus* 50e5-6, 51a5) that "receives all things" (51b) as copies or images (50e2), without "showing its own face" (50e4) in the process.

This would suggest that the Receptacle provides no resistance of its own when being informed by the Forms, indicating that whatever character it was to receive it would receive it in a pure and accurate manner. By analogy, a straight mirror can be called characterless because it will represent its model accurately, i.e. without importing any of its characteristics to the way it depicts it. In other words, the proportion and shape that a reflection of a triangle would have will be due solely to the proportion and shape of the model reflected. By contrast, a crooked mirror will "show its own face" by contributing as a cause alongside the model to the proportion and shape of the reflection, giving rise to an inaccurate depiction.

The Receptacle's lack of characteristics of its own ensures not only that it can receive "all things", but that it does so by letting them appear as they are. This would imply that whatever character a particular might possess, it would do so solely because of their model, i.e. the Form. This conclusion though runs contrary to Plato's emphatic distinction between the Forms and particulars. Forms are described in stark contrast to the

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2 Lee (1964), Mohr (1985).

3 Against this view see Kung (1988).

particulars as being completely insensible and accessible to reason alone (*Phaedo* 65d-66a7, *Phaedrus,* 247c6-9, *Timaeus* 27d-28a), not extended in space and incorporeal (*Phaedo*, 66a, 74b-c, *Symposium* 211a), not in any place at all, and not divisible into parts (*Timaeus* 52a-b, *Phaedo* 78c, 80b-c), and lacking even the conditions for either change and decay (*Phaedo*, 78d-79a, *Symposium* 211a, *Timaeus* 27d-28a).

How are we then to understand the Receptacle's pure mediality, i.e. its undistorting reception of the character of the Forms, in light of the fact that the images are described as contrary to their models? This is the main question that this article proposes to address.

A second, related question, regards the nature of the aspect that binds an image to its model, since they are described by opposite properties.

These two questions can be summarized as follows:

1) If the Receptacle receives without distortion the character of the Forms, what accounts for the fact that the resulting images are characterized by contrary properties to their models?

2) How are we to understand the positive link, or the common character, that makes something be an image of its model, and what exactly accounts for the fact that an image is of one particular model rather than another?

In order to provide an answer to these questions, I will first place my investigation within the framework of some general interpretation of the nature of Forms and particulars and of the nature of participation. For this purpose, I will analyze three of the more popular interpretations belonging to the secondary literature. I will examine in turn the Approximation view of participation, the Unqualified Exemplar interpretation, and the Model-Image view. I will argue in favor of the latter as being the most feasible approach.

Following this, I will use the theoretical framework of the Model-Image view in order to tackle the questions this article has set out to answer. To this purpose I will argue that the Receptacle is not characterless in all respects, but rather only in those respects in which it is to receive a determination from the Forms. Then, I will claim that intrinsic to the concept of image lie two different determinations. The first is the formal aspect, the determination that the image receives from its model. The second is what I have called the medial aspect, and

concerns the characteristics that the medium in which the image comes to be effects on the image. My main argument will be that we can account for the differences that distinguish the image from the Form by way of the medial aspect of the image. I will argue that such properties that belong to the particulars such as extendedness, visibility, composability, and being in flux are due to characteristics intrinsic to the Receptacle, the medium in which they come to be.

Regarding the second question, I will contend that the Model-Image view cannot offer a satisfying account in its present state. I suggest that an analysis of the "being of" relation that binds an image to its model might provide the necessary tools in order to provide an answer to this question.

2. The relation of particular to Form

A good way of demarcating between the ways that the relation between particular and Form has been interpreted throughout the secondary literature is by an author's choice of word when translating εἴδωλον, μίμησις or εἰκών. These terms are all used by Plato as metaphorical stand-ins for the worldly particulars[4], and have been usually translated either as copy or as image. The two words carry with them substantially different implications, though. While the copy suggests that the particular is in some way dependent on the Form, it also implies that 1) it is the same type of thing as the Form and 2) it leaves open the possibility-if not explicitly denied- that it can equal the Form in perfection. A copy key owes its existence in part to a model and can be evaluated by reference to the model, yet it is still as much a key as the model is. When applied to the Forms such a view implies self-predication and thus opens the Theory of Forms to the Third Man Argument of *Parmenides*. As the argument goes, the same way that both model key and copy key, while different in some respects, nonetheless

[4] Plato uses a variety of terms to describe the particulars. Usually usage will vary according to whether Plato wants to underline the common ground between image and original (εἰκών/ὁμοίωμα/μίμησις) or the difference (φάντασμα/εἴδωλον).

have the same property of being a key, so would the Form of Circle and the particular circle both be circles[5].

The alternative view would account for the particular-Form relation through the lenses of the image. While maintaining a relation of dependence to the model, the image does not imply that it is the same way *F* as the model is. As Allen puts it:

> The theory of Forms involves two fundamental doctrines: (a) that the relation between particulars and Forms is that of imitation, of copy to original, and (b) that Forms and particulars differ in degree of reality. The proponents of self-predication maintain that it implies still more: that if *F* particulars and the *F* Itself resemble each other; they must do so in virtue of being *F*. This conclusion is one of almost breath-taking eccentricity. My hands resemble each other in being hands. Do they also resemble the Hand Itself in this respect? Clearly not. For the relation of hands to the Hand is analogous, on Plato's account, to the relation between pictures or reflections of hands and hands. Therefore, if 'the logic of Plato's metaphor' implies that the Hand is a hand, it also implies that the picture of a hand is a hand; which is absurd (1960, 152).

According to this approach we should view the relation of Form to particular along the lines of relations such as those between Socrates and a painting of Socrates, a vase and its shadow, or an event and the retelling of the event through words. Accordingly, an image *F* is not similar to its model in respect to *being F*, the same way that a picture of a cat is not similar to a real cat in being a cat. One of the merits of this position is that it can make sense of participation while avoiding self-

[5] As Patterson (1985, 14-15) puts it: "It is a sufficient condition for a thing's being a *standard* for the type or property *F* that instances of the type are classified on the basis of similarity to that particular *F* in respect *F* [...]. Paradigm cases, standard instances, or perfect particulars are still cases, instances, or exemplars of a given kind of thing, right along their non-paradigmatic brethren. The Standard Yard will itself be one yard long. The standard *F* may be *F* by 'definition;' [...]. It may on these grounds be considered ontologically and epistemologically a different kind of *F* from non-standard *F*. Still it is similar to other *Fs*, and comparable to them in respect *F*."

predication and thus availing the Forms from the Third Man regress, and in consequence offering a more charitable reading of Plato.

3. The particular as copy

Let's look first at the interpretations that view the relation of Form to particular by way of the relation between copy and original.

The Approximation View (A.E. Taylor 1922, W.D. Ross 1951, P. Shorey 1933) argues that Forms are perfect particulars. They have *F*[6] perfectly, while particulars can only approximate, but never equal the perfection of the Forms. According to this view what is essential to the gap that separates Form from particulars is the quality of the properties they possess. The Form of Circle is nothing else then the absolutely perfect circle, and what separates it from all the worldly circles that merely approximate it, is just this perfection. Yet both the Form of Circle and the sensible circles share the property of being circles.

Following Nehamas (1975) in his review and critique of the Approximation View, this type of interpretations relies on the assumption that Plato's main reason for developing the Theory of Forms can be understood on the basis of his background in mathematics and geometry. Because geometry operates with perfect figures that are nowhere to be found in nature[7], where instead we only encounter the merely approximate, Plato had to conceive of a world where they could exist perfectly. More so, Nehamas suggests that this type of interpretation generally assumes that

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[6] "F" is used here as a general stand-in for any property a thing might poses and for which there is a Form, e.g. cold, beautiful, just, horse, etc.

[7] This passage from Shorey (1933, 172-3), captures the spirit of this interpretative direction well: "Experience can never give us the pure mathematical ideas which sensation and perception awaken in our minds. There are no perfect circles or equalities in nature. Yet we do conceive them, and we feel how far concrete circles and equalities fall short of the ideal toward which they strive [...]. We are reminded by the imperfect copies in the world of sense of something that we have seen or known in another state of existence."

> Plato, either consciously or unconsciously, applied this sense of imperfection to objects belonging to ethical and aesthetic contexts. Just as geometrical illustrations are always only approximately and never exactly equal, circular, or square, so beautiful people, just actions, and healthy animals are only approximately and never exactly beautiful, just, or healthy. That is, they could always be more beautiful, more just, or healthier. It is in this sense that the Form of, say, beauty, which is perfectly (namely, exactly) beautiful is like the limit of an infinite series. (Nehamas 1975, 107).

Seeming is also construed along similar lines. For although two sticks might appear to be equal in some respect, on closer inspection, or with the right instruments, it would become manifest that they are actually unequal. Thus, what at first glance appeared as being a perfect instance of equality, proved to be just an appearance of equality, and actually an instance of inequality.

A variant of this interpretation[8] argues that particulars can in fact equal the perfection of the Form, but reserves to the latter the status of unique condition for the hierarchization of the former. According to this view, the uniqueness of the Form is preserved by the fact that it is both an epistemological and an ontological condition for particulars of its type.

One way of seeing the differences between these two versions of the Approximation View is the following. The first ascribes a superior role to the Forms by way of the degree in which they possess properties (perfectly), and thus feels the need to deny perfection to the particulars, so as not to compromise the superiority of the Forms. The latter, on the other hand, identifies the Forms' superiority to their role as epistemological and ontological standards, and thus does not need to restrict particulars from achieving perfection.

What these views hold in common is the fact that they implicitly or explicitly hold that Form *F* and the particular *F* are similar in being *F*, marking the differences between the two in the manner of perfect model to imperfect copy, or standard instance or paradigm and its copy, be it

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[8] I will not be referring to this particular view when talking about the Approximation interpretation. For a more detailed account of this account of the Forms, see Patterson (1985, 13-16).

perfect or imperfect. Such a position ascribes self-predication to the Forms and thus open the Theory of Forms to the Third Man Regress[9].

The Unqualified Exemplar[10] interpretation or the *F* and non-*F* view (Nehamas 1975) while resembling those presented above in that it accepts or involves self-predication, gives a wholly different account of how we are to understand the particular's inferiority to the Form. First off, this interpretation finds the main thesis of the Approximation View untenable. The reason for this is that if particulars only have in an approximate way the property they participate in, then it follows that they also have the opposite property with regards to the same respect and at the same time[11].

If two sticks appear equal, but only approximatively so, on what grounds, Nehamas asks, are we to say that they strive for perfect equality rather than perfect inequality[12]? One of the main characteristics of the *F* and non-*F* view is that it accepts Forms only for incomplete predicates such as large-small, just-unjust, beautiful-ugly, equal-unequal, etc., which are to be distinguished from complete or simple predicates such as man, horse, house, etc. While the former are always said of a thing in relation to something else, the latter don't need any such qualification in order to be predicated of a subject. As such, the possession of incomplete predicates by a particular is wholly dependent both on the object with which it is put in relation, as it is on the context.

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9 For some this is not a problem at all. Owen (1953) for example takes the criticism of the *Parmenides* as proof of a departure from the classic Theory of Forms of the *Republic* and *Phaedo*, and as such places the time of composition for *Timaeus*, on account of the fact that it uses the now defunct terminology of *eidolon* and *paradeigma*, to the middle period.

10 While it is not clear if this view follows either a copy based or image-based interpretation of particulars, I will place it in this category because it allows for self-predication.

11 *Republic* (436b-c) is clear evidence of the fact that Plato did not think this was possible.

12 Nehamas quotes Allen (1960, 178): "A crooked line is not an imperfect instantiation of straight linearity; on the contrary, it is a full and complete instantiation of the kind of crooked line that it is, and the kind is repeatable, though the line itself is not [...] to say that something is deficient with respect to one character is merely an awkward way of saying that it quite fully has another."

Nehamas' reason for restricting Forms to incomplete predicates is quite straightforward. Because contrary predicates such as light or heavy, tall or short, just or unjust, could be said of the same particulars, Plato was said to have

> postulated the Forms in order to show that despite their compresence these properties did correspond to distinct entities, and that the terms associated with them did have distinct, and univocal, meanings. But for this problem to even arise, the same sensible particulars would have to be qualified by contrary properties. And for these particulars to remain the same, there would have to be some properties which those particulars possessed in themselves, independently of their relations to other objects, properties which would allow their reidentification over time[13]. (Nehamas 1975, 166).

If the Approximation view holds that the basis of the inferiority and separation between Form *F* and the particular *F* lies in the imperfection of the property *F* that the particulars possess, the *F* and non-*F* view locates this imperfection not in the incomplete way that a sensible *F* is *F*, but rather in the way that individuals possess *F*.

Particulars can have perfect equality, for example, yet what separates them from the Form of Equality is the fact that their equality is a relative, incomplete and accidental property: relative, because it needs something else to be equal to, incomplete, because equality does not hold in all aspects and in all contexts[14], and accidental, due to the fact that the object can subsist without being equal to anything else. If two stick appear equal in length, their equality in the respect in which it holds is in no way inferior to Equality itself. The difference between the equality of the sticks and that of Equality itself is that the former are equal only with regards to length, for a determinate period of time, they need each other in order to have the property "equal" and can subsist

[13] Nehamas is referring here to complete predicates, or substance sortals. These are predicates that apply in an unqualified manner, such as tree, human, chair, etc.

[14] Two sticks might be equal in size yet not also in width. Also, they may cease to be equal if one of the sticks has a part of it cut off.

even if they somehow lose this property. By contrast, the Form of Equality is equal in all possible respects, for eternity, needs no other thing to be equal to in order to have the property "equal", and because "equality" is its essential property it cannot subsist without it. Accordingly, Nehamas argues that when we find Plato saying that

> sensible objects are only imperfectly beautiful or just, he does not mean that they are approximately beautiful or just. Rather, he means that they are only accidentally beautiful or just, while the Form and its characters possess the relevant property in an essential manner. Notice also that on this approach, not only the Form but also the properties of particulars (the characters) exhibit this perfection. Thus, the properties that particulars possess are perfect copies of the Forms in which these particulars participate. (Nehamas 1975, 109)

For instance, Helen can be both beautiful (in comparison to a monkey) and ugly (in comparison to Aphrodite). According to Nehamas then, it is not that Helen possess beauty approximately—more so than the monkey and yet less then Aphrodite[15]—but because Helen possesses beauty in an accidental manner, its possession is always dependent on the relatum with which she is put in relation[16].

One consequence of this view that Nehamas endorses is that Forms of incomplete predicates are what they are in an unqualified manner, and which allows for self-predication. That means that something like Equality, for example, would not only be an equal thing that is equal in all possible respects (completeness), but it would also be equal without anything else to be equal to (non-relative).

In summary, the difference between the Approximation View and the *F* and non-*F* view can be articulated as follows: while in the first case the individuals have imperfect and only approximate properties, in the other case the individuals have perfect properties, while their

[15] For arguments against construing participation as a matter of degree see Nehamas (1975, 110).

[16] One can ask as to what accounts for Helen's possessing beauty in relation to the monkey and losing that property in relation to Aphrodite.

imperfection lies in the way the particulars *possess* their properties and not with their properties as such.

The criticism that both these views will receive from the image theorists is that they provide a rather uncharitable reading of Plato. Both positions imply self-predication and the regress of the Third Man, consequences that the image theorist argues that it could easily be avoided if the difference that separates particulars from Forms were to be thought of in terms of an ontological difference and not merely in terms of the properties they possess or of how they possess them.

If the image theorists can accept Nehamas' critique of the Approximation view, and also that at times being *F* and un-*F* is indeed a characteristic mark of the sensibles, they will not concede either that 1) there are Forms only for incomplete predicates, 2) that Plato thought of Forms as unqualified exemplars of qualified particulars[17], or 3) that being *F* and non-*F* is the sole mark of the particulars' inferiority to the Form.

Let us now turn to the image theorists' conception of the relation of Form to particular, and to their conception of the characteristic marks that differentiate the two.

4. The particular as image

The Image-model interpretation (A.I. Allen 1960, E.N. Lee 1964, R. Patterson 1985) assumes an ontological distinction between particular and Form. What separates them is not a matter of the degree in which they are *F*, nor of the way in which they possess *F*, but rather, first and foremost, the fact that they are not *F* in the same way. Patterson writes:

> Indeed, Plato's stock examples of images-paintings, statues, drawings, reflections in mirror or water, dream images, songs, images in poetry or prose- are in no way related to their models as copies to standards or as qualified to unqualified exemplars. In

[17] This would have the consequence of suggesting that a thinker the caliber of Plato could have blundered his way into thinking that essentially relational Forms such as Equality or Large could have the property "equal" or "large" irrespective of anything else.

> this case the image *F* is not 'another real *F* such as its model' (the phrase is from *Sophist* 240a9), nor does it resemble its model with respect to being *F*: the reflection of Cratylus in the mirror or on water is not another Cratylus; the black-figure warrior on a vase is not another, only qualified or imperfect, warrior; […] the mirror reflection of a bed is no more a 'real' bed, a worldly participant in the Form of Bed itself, than is the mirror reflection of a horse, since neither fulfills the function or does the characteristic work of a carpenter's bed. (1985, 20)

Starting from this interpretation of the relationship between the Form and the particular, two different positions with regard to the status of the relation that binds particulars to Forms can be distinguished. The first view (Allen 1960, Lee 1964) proposes to give up on any such relationship. Both propose a desubstantializing interpretation of particulars, thought on the model of the relationship between the reflection and the reflected thing. For both Allen and Lee, the power and essential meaning of the metaphor of the image comes to light only when we consider the shadows and reflections and the type of dependence specific to them. They argue that just as a reflection in a mirror is not really something, in any case, not something independent, but rather the effect of the interaction between a thing and a medium, in the same way we do not need a relationship to bind the model reflected to its reflection. In Lee's words:

> I maintain that the very being of a reflection is relational, wholly dependent upon what is other than itself; the original, and the reflecting medium. The gist of my hypothesis, as of his [Allen's], is that because their being is relational, adjectival, dependent, relations to bind them to Forms are neither possible nor required. The `insubstantial image' is not an entity related to a Form; it is the product of a relation-perhaps just the holding of a relation-between something else [the Receptacle] and the Form (1964, 365).

On the other hand, Patterson (1985) argues against reducing all types of images to the completely dependent nature of reflections and shadows,

arguing instead that the substantial images such as paintings, sculptures, or written accounts that Plato mentions, need to be taken at face value. This position will make him plead for maintaining a quasi-substantial character of the sensible world and, therefore, for the need for a relationship that links it to the intelligible one. This relation is described in terms of the images' being *of* its model:

> The positive link that removes image *F* and model *F* from the realm of bare equivocation is the image's being an image of its model [...]. There is thus an extended family of *F*s held together, despite differences of type, by the relation of imaging. I suggest this is an adequate explanation of Plato's speaking of "likeness" with respect to being *F* between two things that are in another sense definitely not alike in that respect. (Patterson 1985, 42)

It is no problem for the image theorist to accept that particulars can achieve perfection, for any type of perfection that an image might achieve would still not make it less of an image. Thus, Patterson argues regarding the perfectly spherical shape in which the Universe was made into by the Demiurge (33b2-7, 34b) that even though "it takes the power of a god to produce a perfect circle in the sensible realm it is beyond the power even of a god to produce a duplicate of the Form within the sensible realm" (94). More so, in the *Republic* (529c-530c), Socrates states that the heavens "must be used as an embroidered model in the study of those realities," and that it would at the same time be "ludicrous to labor over them as if he would grasp the truth of equals or doubles or any other ratio." Patterson suggests that even though the movement and constitution of the heavens were to be perfect as it is the case in the cosmology of the *Timaeus*, they would still only be "visible models of true realities." The mark of their inferiority thus resides not in the sensible circles' imperfection as such, but in their visible character. In other words, the main reason that the visible world is not an object of ἐπιστήμη is not that it provides imperfect or approximate instantiations of otherwise perfect figures, but that it provides visible instantiations as such. The visible circle belongs to the generated phenomena, and because of that all that it tells the understanding is that it is what it is, i.e.

a circle, and not why it is so, or what it means to be so[18]. For the latter accounts one needs to ascend from the generated to the generative source, a task that can be accomplished by reason alone.

So, at least in Patterson's case, we find that the model-image interpretation construes the Forms as abstract essences that act as principles for the generation of the world of becoming, and of the particulars as the concrete, generated instantiations of these essences[19]. More so, there seems to be no obvious point of identity between the Form and its image, given that even if the sensible sphere were perfectly spherical it would still not be "similar to the Form with respect to shape" (ibid.). Patterson makes this point on the basis of a line in *Phaedrus* (247c) where Socrates states that the Forms have neither color nor shape. Another more forceful argument for understanding Forms as abstract essences emerges when one thinks of the relation between the Form of Shuttle (*Cratylus*, 388a-b) and the many different types or species of shuttle that the Form must provide an essence for. If it were that the Form of Shuttle was either a perfect shuttle or a pattern or blueprint for some kind of shuttle, it would not be able to provide the essence for the many different species of shuttle that there are.

From the perspective of the image theorist both the Approximation as well as the Unqualified Exemplar views provide merely accidental criteria for differentiating between Form and particular. As we have seen, some particulars can attain perfection and still not equal the Form in any way. Regarding the Unqualified Exemplar view, Patterson (100) points out that in the *Phaedo* (106d5-6) Socrates talks about the Form of Life, which even though it has a contrary, i.e. Death, is still a complete predicate. This amounts to a critical objection to this view[20].

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[18] See Cornford (1997, 24).

[19] "[…] what Plato requires as standard is an abstract intelligible nature or essence which can be exemplified in various ways while providing a criterion of excellence for any and all of its sensible namesakes. Thus, the Form of Equal itself has no shape, weight, size, speed or age, but is participated in by various sorts of equals;" (Patterson 1985, 109).

[20] This, Patterson writes "provides one more reason for not supposing Plato's Forms were, to begin with, unqualified exemplars of incomplete predicates, differing from sensibles by performing the impossible- by providing examples of largeness, equality and so on without being large or equal in relation to any relatum" (100).

Our next step is to see how the Image Theory construes the differences that mark the distinction of image to Form.

5. Differentiating between Form and image

Let's begin by looking at some of the passages that Patterson provides in order to argue for the inferiority of the sensibles.

In the *Phaedo*, Socrates fends off Cebes' concern that a man's soul might disperse at the moment of his death, by asking what kind of thing is likely to be dispersed, i.e. what nature a thing must have in order to be able to succumb to destruction. The argument goes that if the soul is more like those realities that themselves do not run the risk of decomposition, then the soul is safe. What follows is an enumeration of the characteristic marks that separate the corporeal from the Forms: at (78c) the property of compositeness is attributed to the corporeal, which in turn leads to the possibility of decomposing or destruction, while the Forms are described as non-composite and thus indestructible. The two are further equated with being in flux and being perfectly stable, respectively. At (79a) the corporeal is said to be visible and perceptible while the Forms invisible and accessible only to reason. At (79d) the Forms are contrasted to the corporeal realities and described as "pure, ever existing, immortal and unchanging." Lastly, (80b) recapitulates what has been said so far:

> Consider then, Cebes, whether it follows from all that has been said that the soul is most like the divine, deathless, intelligible[21], uniform, indissoluble, always the same as itself, whereas the body is most like that which is human, mortal, multiform, unintelligible, soluble and never consistently the same.

[21] One interesting aspect of this enumeration is that while at (79a) the corporeal was said to be visible and was contrasted to the Forms' invisible and intelligible character, at (80b) intelligibility is contrasted directly with unintelligibility, omitting the visible-invisible pair. This suggests to me that for Plato visibility is not an accidental trait of the unintelligible, and neither invisibility for the intelligible, but rather that they are essentially connected.

Turning to Diotima's account of the nature of Beauty itself, we find that:

> Nor will the beautiful appear to him in the guise of a face or hands or anything else that belongs to the body. It will not appear to him as one idea or one kind of knowledge. It is not anywhere in another thing, as in an animal, or in earth, or in heaven, or in anything else, but itself by itself with itself. (*Symposium* 211a-c)

The fact that the Forms are not in something else should be put in relation with *Phaedo* (66a) which states that knowledge can be achieved only by reason alone which, by "using pure thought alone, tries to track down each reality pure and by itself." We can suppose that the alternative to "pure thought alone" tracking down "reality pure and by itself," i.e. the study of the nature of things through the senses and by way of the objects of the sense, can be called impure for no other reason than by the admixture of the perceptual or corporeal element alongside the Form in the constitution of the particular[22]. The introduction of the Receptacle in the *Timaeus* as the corporeal counterpart to the Forms in generating the particulars sheds a revealing retrospective light on these passages.

Timaeus echoes both Diotima's description from *Symposium* and Socrates' from *Phaedo* when he states that:

> Since these things are so, we must agree that that which keeps its own form unchangingly, which has not been brought into being and is not destroyed, which neither receives into itself anything else from anywhere else, nor itself enters into anything else anywhere, is one thing. It is invisible—it cannot be perceived by the senses at all—and it is the role of understanding to study it. The second thing is that which shares the other's name and resembles it. This thing can be perceived by the senses, and it has been begotten. It is constantly borne along, now coming to be in a

[22] We find a similar vein of thought in *Republic* when Socrates describes dialectic as "whenever someone tries through argument and apart from all sense perceptions to find the being itself of each thing and doesn't give up until he grasps the good itself with understanding itself" (*Republic* 532a-b).

> certain place and then perishing out of it. It is apprehended by opinion, which involves sense perception. (*Timaeus* 52a-b)

Moreover, with regards to the image Timaeus states that:

> Since that for which an image has come to be is not at all intrinsic to the image, which is invariably borne along to picture something else, it stands to reason that the image should therefore come to be *in* something else, somehow clinging to being, or else be nothing at all. (52c-b)

In the *Republic* (479a-d) Socrates describes the condition that befalls all particulars that participate in opposites. "So, with the many bigs and smalls and lights and heavies, is any one of them any more what we say it is than its opposite? No, each of them always participates in both opposites. Is any one of the manys what we say it is, then, any more than it is not what he says it is?" This, in turn is to be compared to the Forms' being what they are without any admixture from its contrary.

Besides these characteristic differences that distinguish Form and particular, there is still the matter of the image being *of* its model. This fact is expressed by Patterson as a kind of double dependence of the particular to the Form "as model for making and as formal *aitia*" (92). Trying to capture the same point, Allen writes:

> Particulars are named after Forms because Forms are their causes. To say of anything that it is *F* is to say that it depends for its existence upon the *F*, that in virtue of which *F*-things are *F*. But the *F* is not merely a cause; it is an exemplary cause. Particulars not only depend upon it; they are resemblances of it, as reflections are resemblances of their originals. Like reflections, they differ in type from their originals; they share no common attribute; and yet they exhibit a fundamental community of character. (1960, 160)

Neither account explains too much when it comes to how we should understand the nature of the relation that binds model and image[23]. I

[23] I shall, for the present purpose name it the "being of" relation.

believe we can at least say the following things about it. First of all, the mode of being of the image is that of being *of* something else. This *of* which references an *other* affords the image three main aspects. a) Its identity; by being named after that which serves as its model. b) Part of its character; the model also is responsible for part of the characteristics of the image, e.g. part of the aspect of a painting of a cube will be due to the cube itself. c) Its evaluative model; the model acts as an evaluative standard from which the accuracy of the image's depiction is judged[24].

To sum up, the image theorist draws on the following passages in his account for the difference of particular to Form: 1) The particular is corporeal and visible, the Form is incorporeal, invisible and shapeless. 2) The particular is open only to perception and opinion while the Form is invisible, intelligible, and the sole object of knowledge. 3) The particular is spatial extended while the Form is not in space. 4) The particular is compounded, the form is simple and uncompounded. 5) The particular is liable to decomposition and change[25], the Form does not have the necessary properties for neither change nor decomposition. 6) The particular is in[26] something else, the Form is in and of itself.

This is as far as the list goes when it comes to strictly textual references. From these passages the image theorists conclude that: 1) The particular is concrete, the Form is abstract. 2) The particular has or images some nature, while the Form is the nature imaged, in the sense that it is what it means to be *F*, and not another an instantiation of *F*. 3) The particular images or instantiates something else, the Form is the abstract nature that is imaged or instantiated. 4) The particular has either

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[24] As Cratylus (432a-d) shows the image must always depict its model in a restricted manner. This implies that an image's accuracy will always be judged in the relevant respects in which it purports to depict its model. For example, one should not say of a picture of an apple that it is inaccurate because it is a two-dimensional depiction, while the apple is three dimensional.

[25] This is true even of immortal beings such as the gods that populate the cosmos and the cosmos itself. At *Timaeus* (41a-c) it is said that the Demiurge's good will accounts for their everlastingness. Thus, the everlasting object, because it is sensible and compounded, has in itself all the conditions necessary for change and destruction.

[26] Recall that particulars reside in the Receptacle, while the Forms "neither receives into itself anything else from anywhere else, nor itself enters into anything else anywhere" (52b-c).

quasi-substantial being (Patterson) or is an insubstantial being (Allen, Lee), while the Form fully is.

Coming back to the positions we have discussed so far, we can resume them the following way:

1) The Approximation view holds that the particular equal and the Form of Equality are both equal things. What differentiates them is that the latter is perfectly equal while the former only has equality in an approximative manner. Furthermore, it is because of this status as a perfect particular that the Form takes the role of ontological and epistemological condition of the other.
2) The Unqualified Exemplar view also holds that both the particular equal and the Form of Equality are equal things. What differentiates them is the fact that the Form of Equality is equal in an essential manner while the particular is equal only in an accidental and thus relative and incomplete way. Contrary to the Approximation view, the equality that the particular possesses is in no way inferior to that of the Equal itself. Its mark of inferiority comes solely from how it possesses it. What makes the Form of Equality an epistemological and ontological standard is that by being equal in a complete and eternal way it provides a) the criterion by which to judge particulars as equal in some respect, and unequal in another respect, and also b) an eternal and unchanging Equal model that can be copied, i.e. participated in, by the particulars which thus become equal in the way open to them.
3) The Model-Image view holds that the equal *qua* particular thing and the Form of Equality are not both equal things. Rather, one is the abstract nature of equality and the latter is its sensible and corporeal instantiation. What differentiates the two is the manner in which they are F, one being an abstract essence, the other its sensible image. According to this view we do not need any common property that both model and image must have in order for the image to be of its model. The argument goes that since the model is an abstract essence that the particular images 1) they do not share, or need not share any common properties and 2) their connection is guaranteed by the images' being of that model. The model provides

> the ontological condition for the image by informing the Receptacle, thus generating the image. It also provides an epistemological condition by being the formal cause for the image.

I believe that the Image Model theory provides the best account of Plato's Theory of Forms. The other two views proved not only to be untenable when confronted with the material that Plato furnishes in his writing, but they also provided a rather uncharitable interpretation, given the fact that both, implicitly or explicitly, imply self-predication. The Image Model theory instead was able not only to make sense in a coherent way of a larger amount of material, but also provided a way for avoiding self-predication. This is not to say that it would not benefit from a clearer analysis of the *being of* relation, and also from a more convincing account of the relation that binds the image to its model. As we shall see in the next part of this article, these shortcomings will become obvious when trying to give a clear articulation of the communality between the particular and its Form, a communality that is essential in accounting for the reason that an image is of some model rather than another.

In my upcoming exposition of the problem that *Timaeus* rises I will place my interpretation in the general framework of the Image Model theory.

6. The Receptacle, Images and Forms

We can now go back to our original question. At the beginning we asked how we should understand the fact that if the Receptacle is a pure medium in which the Forms leave their respective marks as images, these images nonetheless possess completely different characteristics from their models.

To begin let's first take a look at how the Receptacle is described by Timaeus:

> We also must understand that if the imprints are to be varied, with all the varieties there to see, this thing upon which the imprints are

> to be formed could not be well prepared for that role if it were not itself devoid of any of those characters that it is to receive from elsewhere. For if it resembled any of the things that enter it, it could not successfully copy their opposites or things of a totally different nature whenever it was to receive them. It would be showing its own face as well. This is why the thing that is to receive in itself all the elemental kinds must be totally devoid of any characteristics. (50e)

Timaeus insists that if the Receptacle "is to receive repeatedly throughout its whole self the likenesses of the intelligible objects, the things which always are—if it is to do so successfully, then it ought to be devoid of any inherent characteristics of its own." The "mother or receptacle of what has come to be" and of what is visible should be thought of as an "invisible and characterless sort of thing, one that receives all things" (51a).

We can divide Timaeus' description in two parts. The first concerns the role of the Receptacle: to receive all things, and to receive them successfully, i.e. without showing its own face. The second provides a description of the character the Receptacle must have in order to achieve its role: it must be "devoid of the characters that it is to receive from elsewhere," it should not resemble any of them, and also it must be an "invisible and characterless sort of thing."

If someone were to use a perfume base[27] that had its own specific smell, say "x," and wanted to imbue it with the perfume essence "y," the resulting effect would not be "y," but some sort of combination between "x" and "y." The perfume base would thus "show its own face" in the end result. Because the perfume base had a prior determination as to smell the procedure failed, it has produced something different then

[27] This is one of the comparisons used to describe the Receptacle. Throughout (49-52) the Receptacle is compared to a receptacle (49a6), to a wet nurse for becoming (49a6), to gold in relation to the multitude of shapes that the gold can take (50a-6), to a mother in relation to the Forms taken as the father, and to the images as their child (50d3-4), to a neutral base for perfumes (50e8), and to a shapeless and soft material to be imprinted upon (50e11). Also, at (52b1) it is described a "fixed state for all things that come to be," and at (52e6) it is compared to a winnowing-basket.

what was intended. As a consequence of this, if someone were to use the resulting perfume as an indication of the smell of the perfume essence "y," he would be misled.

Yet, it was not the fact that the perfume base had a prior determination that compromised the procedure, but that it had the relevant kind of determination. A perfume base can be determined as to both shape and color and in no way affect the odor it receives. The same way, the material for imprinting that Timaeus brings up as an analogy for the Receptacle (50e11) can be determined as to odor and color and not affect in anyway the imprinting process. A thing can thus be called characterless in some specific aspects while at the same time be determined in others.

Returning to (50e) where Timaeus states that if the Receptacle resembled "any of the things that enter it, it could not successfully copy their opposites or things of a totally different nature whenever it was to receive them," I believe this passage should be read the following way. The Receptacle cannot be determined in itself[28] with regards to one of a pair of contraries. If it were cold for example, even though it could receive hotness, it would do it by changing its own coldness, i.e. as a thing heating up, and would modify the character of what it received, thus "showing its own face" in the process. With regards to "things of a totally different nature" I take this to refer to things that don't have any contraries, e.g. circles, triangles, trees, houses, etc. If the Receptacle resembles a circle it would not be able to receive the nature of a triangle. In the case of geometrical shapes, the Receptacle would have to be more like a plane. Yet, and this is a further consequence, it still could not resemble a plane for then it could not receive three dimensional things, or if it were only a plain it could not receive movement[29]. However, I suggest that the same way the plane is the condition of possibility for the manifestation of all two-dimensional geometrical shapes, without being their formal cause, the Receptacle can be understood by analogy as a plane that is general enough in order to receive "all things" (51b1).

It follows that if the Receptacle receives all things, and is indeed characterless in the relevant aspects so as not to "show its own face" at

[28] Rather, as Timaeus show at (50b7-c1), all the determinations the Receptacle takes never affect it as such, for "it does not depart from its own character in any way."

[29] For movement is not implied by the concept of plane alone.

any time, then whatever may appear in the Receptacle 1) does not originate from the Receptacle, the same way the smell that a pure perfume base receives does not originate from the perfume base, and 2) whatever character we may encounter imprinted in the Receptacle is exactly the same in some relevant respects to the character of the Form[30], the same way that the sigil in wax is identical with respect to form to that of the stamp.

Yet such conclusions can hardly be accepted at face value. It would mean that whatever is accessible to us in the domain of the sensible world, i.e. whatever is accessible by way of perception and also the perceptible as such—recall that the Receptacle was called the mother of the visible, yet itself invisible (51a)—would be a reduplication of the character of the Forms. By analogy, if whatever smell may appear in an odorless perfume base should be taken as the exact same odor of some perfume essence, so it should be that whatever character may appear in the Receptacle should be taken as the exact character of the Form.

If indeed the Receptacle is a pure medium that receives without any interference of its own the character of the Forms in the shape of images, it follows that by studying these images one could get an accurate account of the Forms, in the same manner in which one would use a mirror reflection to study the thing itself. But this runs against the fact that the particulars were described as scattered or extended, visible, in flux, corporeal and decomposable, as opposed to the Forms' lack of any spatial characteristics, invisible, always the same, intelligible and indecomposable. How are we to make sense of the Receptacle's pure mediality if the images it carries seem to misrepresent their model in almost all conceivable ways?

I will try and answer this question first by stating that images can misrepresent their model only in one way, i.e. in the respect in which they aim to represent it. If someone were to say that a picture of Athens misrepresents Athens because it is two dimensional, only a few centimeters across, and so on, we could rightly say that he does not understand what a picture is. In using images, we usually eliminate the

[30] I follow here Lee (1964) and Mohr (1985) against Cherniss in taking the images as a byproduct of the Forms and the Receptacle and not as a fourth kind of independent thing besides the Forms, the Receptacle and the phenomena from within the Receptacle.

idiosyncrasies of the image as such from the representational content of the image. We do so by taking into account what type of image we are dealing with, a painting, a shadow, a spoken account, a statue, etc., and we exclude those characteristics that are specific to the medium of representation in judging whether it distorts its model or not. Besides, even though these types of images have more or less in common with their model, for example a painted statue of Socrates would be three dimensional just as Socrates is, while the painting would not share this property, we would not judge their accuracy[31] on the basis of this general property sharing. Rather, we would reserve accuracy for the measure in which each image achieved sameness in the respect it set out to do so: the sculpture with regards to proportion, color, and shape in a three dimensional medium, the painting with regards to proportion, color and shape in two dimensions, while the shadow only with regards to proportion and shape.

I believe the same to be the case with the Receptacle. Spatiality, scatteredness, visibility[32], compositeness, flux, all this constitute essential determinations of the Receptacle that it passes on to the image, and that should be excluded from evaluating its truthfulness, the same way we exclude a photo's two-dimensional mode of representation when judging whether or not it accurately represents its model. As we have seen, there is no problem in ascribing certain determinations to the Receptacle[33], as long as these determinations do not interfere with the way it receives and properly represents the characters of the Forms.

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[31] In this case we would probably say that the medium of sculpture is potentially a more informative medium when it comes to fully depicting people than that of painting or of shadows.

[32] I believe that the Receptacles' invisibility (51a) is different from that of the Forms. I take it that we can conceive of invisibility in two way: a) as the un-visible, the way numbers, sounds, or abstract Forms are not visible, and b) as the undetermined with regards to color, and thus conductive of color, the way Aristotle's diaphanous medium is invisible. I suggest the latter is the case for the Receptacles' invisibility. This reading follows the spirit of the analogies with the material for imprinting and the perfume base. Both can be called unshaped or odorless in the sense of not yet having any determination with regards to shape or odor, and not in the sense in which they could not be determined in principle in these respects.

[33] This is not to say that the fact that the Receptacle can have determinations without participating into any forms is not puzzling.

This can be seen more clearly if we think of the image as having two faces. One is the face it purports to show, the face of its model, in the way that it is able to show it. We can call this the formal aspect of the image. The formal aspect amounts to the formal cause of the image. On the other hand, an image also has the face of the medium to which it belongs. We can talk about images in sculpture, painting, reflections or words. Each image will bear the specific determinations of the medium in which it arises: an image of Socrates in the medium of painting will be different from one in the medium of words not because of the model they represent, but because of the type of representation they are. Let's call this *the mediatic aspect of the image*. Thus, the mediatic aspect determines the way in which the formal aspect can manifest in an image. To illustrate, think of the different ways in which a red apple is imaged in a drawing, a painting, in its shadow, in a sculpture, as a mathematical description, in logos, or in memory. Each mediatic aspect opens up certain possibilities of imaging while at the same time closing up others. As we can see, in this case the images differ not because of the model they represent, but because of the specific medium in which they image their model. More so, because one has had contact with the apple itself, he can 1) call the rest mere images of the apple and 2) identify them as images of the apple.

Furthermore, in none of the cases mentioned above does the model determine in any way the structure of the medium of representation. The model and the medium are thus independent of each other, while the image is in different ways dependent on both. I suggest that the Receptacle should be interpreted as constituting the mediatic aspect of the particulars taken as images, while the Forms constitute their formal aspect. It should come then as no surprise that the image of the Form also bares distinctly different properties then the Form itself. These are due to what I have called the mediatic aspect of the image.

Now that the images' difference from the Forms has been accounted for, we must also try and give an explanation of the relation that connects them. First, it should be recalled that the Forms have no shape, are not spatial and are invisible. Thus, spatial or in any way visible triangles —even if only to the mind's eye—will not resemble the Form with respect to their spatiality or visibility. A particular triangle's

or circle's character of visibility and extendedness is due to what I have called the mediatic aspect, for it is the Receptacle that offers images their spatial and visible character. Yet what accounts for the way in which they are extended in space, and thus for us calling them triangle rather than circle, is the formal aspect, in this case the abstract Form of Circle and Triangle, respectively[34].

Yet, from this point on, I believe that the Image Theory runs into some trouble. It becomes quite difficult to understand just how we could read the relation of imaging into that of abstract essence and concrete instantiation. For Allen, the particulars "like reflections, differ in type from their originals; they share no common attribute; and yet they exhibit a fundamental community of character" (1960, 160). This is followed by Patterson considering that "the positive link that removes image *F* and model *F* from the realm of bare equivocation is the image's being an image of its model." (1985, 42).

If all these accounts are illuminating for the way Plato conceives of the relation between Form and particular, they do nothing to explain how this relation works. Patterson insists that the image in no way resembles the Form with respect to *being F*, but that it resembles it with respect *F*, as a different sort of *F* (an image *F* than a real *F*). All that these passages say is that the very essence of an image is to be *of* something else, but they do not reveal on account of what an image is *of* that model. In other words, it is a description of the relation of imaging without an account of the necessary and sufficient conditions for the relation to take place.

The image metaphor would suggest that an image, while of some model, is so by way of representing some aspect of the model. In other words, the image always takes on some aspect of the model and wears it, as it were. Yet, if an abstract essence does not itself possess the properties that are to be instantiated by the particular, it is not at all clear

[34] Mohr (1985, 88) makes a similar distinction with regards to the particulars of *Timaeus*, albeit for a different point: "On the one hand, they are in flux; on the other hand, they are images of Ideas. Insofar as the phenomena are in flux, nothing whatsoever may be said of them. But insofar as they are images of Ideas, they may be identified according to kind." On my interpretation the particulars' "being in flux" is just another characteristic that is due to the mediatic aspect, i.e. the character of the Receptacle.

how the relation of imaging should apply in the case of abstract to concrete instantiations. If a painting images its model by way of reproducing its aspect in color and shape, what is there to reproduce in the case of an abstract essence? The essence surely does not itself have the properties of the things it is an essence of according to the image theorist. The Form of Triangle is not itself a three-sided thing, but it is the essence of all three-sided figures. In the words of Patterson "it just is what it means to be *F*." He suggests that in order to accommodate *logos* in the order of images we need to understand the semblance that connects image to model in terms of correctness. If we can say that a painted tree shares with the real one the same color, shape and proportionality, and in this context interpret semblance between two objects in terms of sharing some identical properties, we cannot give the same explanation in the case of a spoken account of the tree and the tree itself. What do the word "red" and the color red have in common? It is for this reason that Patterson calls for the semblance that Plato so often speaks as connecting model and image to be interpreted as correctness. Yet his analysis[35] neither shows how this could be done or even if it is possible.

As the interpretation stands at the moment, I believe that it can offer no clear answer as to what determines, for example, whether a particular circle is an image of the Form of Circle rather than of the Form of Triangle. All it can do is say that the image circle is indebted to the Form of Circle for being what it is, but not also what this debt amounts to, i.e. what is it exactly that it receives.

7. Conclusion

To summarize, the Image Theory interpretation of Forms took us halfway in answering how it is that the images the Receptacle holds are at the same time radically different from their models and also pure expressions of their character. First, I have argued that the Receptacle need not be absolutely characterless—if such a thing is even possible—in order to receive all things, and receive them as they are, but that it only

[35] For the relevant passages see Patterson (1985, 110-113).

needs to be characterless in the domain to which the things it receives belong. Then I have argued that all images have a double determination, a) their formal determination, i.e. the way they are determined by the model they are of and b) their medial determination that comes about from the specific medium in which they image their model. Following this, I have ascribed all the characteristics that differentiate the particular from the Form to the medial determination of the Receptacle. Finally, I have pointed out the difficulties that an abstract account of Forms runs into when trying to make sense of the positive relation between Forms and particulars.

Even though I don't believe that in its present state the Image Theory can give a satisfactory answer as to how we are to understand the positive relation between particular and Form, this does not spell the end for such an approach. It could be that an analysis of the *being of* relation that binds image to model may reveal a feasible way of accounting for what it is that the model invests the image with. First off, the image is not connected to its model by an act of conventional reference. Nor is the image connected to the model the way some effect is connected to its cause, both completely distinct things, yet connected by the causal relation. The image makes us think of the model because it has something that belongs to the model and reminds us of it. Their connection lies in the fact that the model invests the image with one of its distinguishing characteristics. What is the nature of the thing invested, and how are we to understand it in the context of Forms and particulars, where Forms are taken to be abstract essences? I believe that the Image Theory has yet to provide an answer to this vital question.

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RESISTING THE PARADOX OF INCREASE

CONSTANTIN PRICĂ[1]

Abstract: The fact that an object gains a new part, when before it didn't have it, is paradoxical. I argue that a holistic approach that considers change, defined as both increase and decrease of parts, can resist this unwanted implication. The approach requires understanding objects within the context of a world, be it finite or infinite. In such a world, objects can remain themselves even after exchanging parts. The primitive notion of essence (following in the tradition of Kit Fine) is central to this outcome. I argue that the actual world is similar to worlds that permit change, despite some concerns regarding vagueness.
Keywords: The Paradox of Increase, The Growing Argument, material constitution, essence

Introduction

The aim of this paper is to show one way of resisting the paradox of increase. Some metaphysical commitments are required, but I consider them by and large uncontroversial, especially because they can be developed in accord with physical discoveries and theories. Other ways to resist or even solve the paradox of increase have been proposed[2]. I will not go into them; the direction of my approach is quite different. Specifically, my approach is holistic and considers increase, decrease and more generally change only inside a finite or infinite world of

[1] PhD candidate, University of Bucharest. Contact: const.prica@gmail.com
[2] Olson (2006) is a comprehensive study of the controversial solutions (as considered by the author) to the paradox.

objects. This is in contrast with a restricted formulation of the paradox in terms of a number of objects[3] and no context.

I believe that, just as motion is possible even if Zeno's paradoxes suggest otherwise, change is possible and real in the actual world even if the paradoxes of increase and decrease would try to convince us otherwise. Trees, cats, humans, etc. do undergo change and are themselves throughout their lifespan. Real change is contrasted to apparent change, existing only at the level of language. I find it preferable to account for change at the metaphysical level of existence, over a piecemeal language analysis of propositions that contain change-like verbs[4].

I will start with the restricted formulation of the paradox, and then go on to expand it in two ways. First, by introducing a background (call it "world") to the objects, and second by introducing the opposite action of decrease among the objects. I will then try to show that objects inside a world, be it finite or infinite, can change (defined as the successive increases and decreases of parts). The notion of essence, understood as primitive[5], has an important role to play in how objects can be identical even if they change. At the end, I will tackle vagueness, and why it isn't metaphysically worrisome outside of some special cases.

1. The Paradox of Increase and the Paradox of Decrease

The Paradox of Increase or the Growing Argument, the name under which it is known from Antiquity[6], simply states that adding a new part to an object is impossible. A straightforward way of exemplifying this can be found in Chisholm (1979) and it involves an object A to which we attach B.

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[3] Olson (2006, 402-403) presents the puzzle as a *reductio ad absurdum* argument in six steps, starting with only three objects: A, B and C (identical with A at the start).

[4] Olson (2006), chapter 4, discusses how language can be made to account for change when real change is impossible.

[5] I will follow Fine's notion of essence as presented in "Essence and Modality" (1994a) and "Senses of Essence" (1994b).

[6] Rea (1995), note 8, presents the most likely origins of the Growing Argument.

> At the earlier time, t_1, A and B were separated; at the later time, t_2, they are conjoined. But what object became bigger? It was neither A nor B, for these things remained the same size they were before. And it was not AB for AB did not exist until A was joined with B. That is to say, AB did not have two different sizes, a smaller one at one time and a larger one at another. (Chisholm 1979, 158)

The assumption to note is that A and B stand for any objects or things[7] whatsoever. Let A stand for a heap of sand, upon which I add more sand. Does the original heap grow or a new heap with more sand in it replaces it on the table? Let A stand for a model ship without a figurehead, and B for the figurehead. Does attaching the figurehead B to the ship A make AB come into existence, or A, the ship, "already had B as a part then and we merely changed it from a disconnected or "scattered" object (like an archipelago) to a connected one" (Olson 2006, 391)?

A person being identical with its body is less controversial than the identity of the body with the sum or collection of molecules from which it is made of. Because with the next logical step, very reasonable, that a collection of molecules/particles is different after removing/adding or replacing molecules, we arrive at the conclusion that the person disappears after said (minor) change. Outside a deflationary view of reality, that has only the basic particles existing, such a general conclusion rings false.

Moral responsibility, in a certain sense, becomes a joking matter. One of the first formulations of the puzzle of increase is, according to (Rea 1995, 529), that of the comic playwright Epicharmus, and it involves a person wanting to collect a debt. The debtor argues that he isn't the same person as the one who contracted the sum of money and therefore shouldn't pay the sum back[8].

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[7] One way of attacking the argument is that B is not an object, but simply a detached part, that doesn't have existence outside of A. Peter van Inwagen in "The Doctrine of Arbitrary Undetached Parts" (1981) discusses this approach.

[8] The debtor isn't the same because his atoms or molecules have changed. Thanks to a reviewer for pointing out that there are other ways of explaining why the debtor isn't the same, such as different time slices that compose him.

Removing a part of an object is deemed impossible as well. This is the paradox of decrease[9]. To exemplify it, take object A that has two parts B and C. After removing B, does A still exist? It seems to be identical now with C, even though before removing B, C didn't have clear persistence conditions or an identity. Do note that what was previously a connected object (A at the start) is now simply a scattered object, by having its two parts simply at a distance from one another. To emphasize further the absurdity of impossible decrease: the action of plucking a hair from your head annihilates or scatters you.

The two puzzles express the impossibility of change. Olson calls this "the doctrine of mereological constancy" and defines it as: "Necessarily, if x is a part of y at some time, then x is a part of y at every time when y exists" (Olson 2006, 392). Object cannot replace their parts, because they would need first to remove a part, and second to add a new one.

Mereological constancy or essentialism[10] goes against common intuitions and beliefs regarding human beings and inanimate objects. I've grown over a period of 32 years, even if all of the atoms that compose me have been replaced during this time. I have had the same parents even after they've had numerous haircuts over the years. I still use the same laptop even after replacing its hard disk drive. I can go on with similar examples.

The picture seems grim, but I will try to show that it isn't. The first move is to specify a context for change.

2. Finite and Infinite Worlds

Let's consider a collection of objects and call it a world. All objects are made of parts, and any parts can be connected or scattered[11] between the

[9] Some other common names for it are: The Amputation Paradox (Olson 2006, 392), The Dion/Theon Puzzle first formulated by Chrysippus according to Sedley (1982), The Tibbles/Tib puzzle first published in Wiggins (1968).

[10] Olson argues for a difference between the two in Olson (2006, 392-393). I don't consider this to have great impact on my approach.

[11] So as to avoid the usage of add or remove.

objects. Parts can be infinitely or not disconnected. If they are an infinite number of objects (or parts) the world is infinite, otherwise it is finite[12].

The first important assumption, that I consider reasonable, is to consider both adding and removing of parts together. In an important sense, they can cancel out[13].

If the two actions are equally likely to happen in a finite world, then that world is static. Therefore, change is a localized property of the world, but globally the two puzzles are satisfied. If they are not as likely to occur, change remains localized, but the world either falls apart or unifies into a single object.

For an infinite world, change is also at least a local feature. The type of infinity the world exhibits[14] is a new dimension to be considered alongside how likely objects are to add or subtract parts. But, all in all, the same outcomes as for a finite world can be described for an infinite world as well[15].

Considering increase and decrease together, within a collection of objects, is just the first step towards an account of change. Identity criteria are required for objects, so that they survive increase and decrease in the world (be it finite or infinite).

3. Criteria for Object Identity

Consider a finite world with only two objects, made up of only two parts: A_1A_2 (A) and B_1B_2 (B). By switching the parts around, the possible combinations, not considering a unique order, are: A_1A_2, B_1B_2, A_1B_1, A_1B_2,

[12] Think of the elementary particles (i.e. electrons, neutrinos, etc.) as the building blocks of a world not infinitely divisible.

[13] A helpful analogy would be a game of Lego, where the player is trying to construct an object by adding and removing pieces. Because the two actions have opposite effects, the player cannot simply add some piece and then remove it afterwards. He wouldn't advance in any real way. He needs some succession of both adding and removing of pieces to form an object.

[14] I will assume an infinite world to have only a countable infinite number of objects and be made of elemental particles that aren't divisible further.

[15] The infinite world has its parts or objects from the "beginning", there is no way to infinitely generate parts or objects from nothing.

A_2B_1, A_2B_2. One example of grouping of replacements that has both A and B survive change is: A (A_1A_2, A_1B_1, A_1B_2), and B (B_1B_2, A_2B_1, A_2B_2). Any of the other groupings also work as criteria of object identity.

In finite worlds, defining identity as an exhaustive list of possible parts replacements is easy. In a sense, objects are no more than these replacements. In an infinite world, the notion of essence, understood as primitive[16], can play a similar role. Some replacements can make an object not be itself (lose its identity), while other replacements will have the object essentially be the same at the end. Determining which replacements are part of the essence of an object is not metaphysically relevant now[17]. What is important is that essence draws a sinuous line in the sand between some that are identity-preserving and some that are not. In the case of abstract objects, such as sets, the demarcation is quite clear. The singleton S cannot suffer increase or decrease and still be itself[18]. In the case of human beings, their hair color or hair length are decisively not essential. Because the body is made of atoms, losing some atoms, or replacing them with other atoms, is possible in some cases[19]. Living things are fuzzier than inanimate objects in regard to their essential parts.

Important to note that essence as a criterion for identity across change is informative only in the context of world. When considering an object outside of any context (the original formulations of the paradoxes), its essence is at most its haecceity. And therefore, it is just a way to ignore the problem of change. Growth and change in a one-object world are nonsensical notions.

The picture sketched so far, of how objects suffer change, still needs some work. First, if essence is only internal to the objects themselves, change can be superfluous. For example, consider a world made up of self-sufficient objects. Second, change can seem *ad hoc*, even if present. This is most apparent in a finite world, as we can exhaustively enumerate all the possible configurations between object parts and

16 In the tradition of Fine (1994a, 1994b), Correia (2006), Lowe (2008).

17 It is of course epistemologically desirable.

18 I consider all sets to be essentially incapable of change, more exactly of adding and removing of parts.

19 The atoms that form the head are intuitively more important than others in the body.

allocate them (*ad hoc*) to one object or another. In an infinite world, determining which configuration of parts belongs to which objects can be quite simple as well[20].

Both observations, that essence isn't sufficient or necessary for change inside a world, are hard to dismiss. A static world, where change isn't present, and a world without objects, just configurations of parts, are both in agreement to the paradoxes of change. The paradoxes shouldn't be dismissed, just resisted as they don't have any power inside other types of worlds.

4. Fleshing out Essence

Can any object share parts with any other object? In the actual world, trees, tables and humans don't exchange their atoms with trees, tables, or humans on the other side of Earth, or rocks on the Moon, or underwater ice lakes on Mars. Essential for objects, in general, is to not get scattered[21], to have a certain spatial and temporal continuity. This continuity gives objects a certain lifetime.

The frequency of exchange of parts characterizes one aspect of an object's lifetime, that of temporal continuity. A higher degree of exchange of parts translates into greater uncertainty of object configurations. A low degree of exchange makes objects rigid. Of course, the exchange of parts need not be investigated at the scale of the world and can be localized to regions of the world (certain sub-collections of objects).

The other aspect, the spatial uniformity, is characterized by the average distance between any two objects that exchange parts. A

[20] Consider the natural numbers, and distinguishing between odd numbers and even numbers.

[21] Collections of coins or stamps are examples of objects that can get scattered. By adding a new coin to the collection, does the collection grow or get replaced with a collection that has more coins? I think the former option is true. After all, essential to the collection is not what is essential to the set of coins that constitute it. A collection of ancient roman coins can be defined as the result of the activity of gathering together, with the purpose of preserving, all the roman coins created. The set of roman coins, like any other set, has its members essentially, and therefore cannot survive growth or decrease.

scattered world has objects from afar exchange parts. The highest value means any object can exchange parts with any other object. Similar with time continuity, spatial uniformity need not be investigated only at the scale of the entire world.

Spatial uniformity and temporal continuity are criteria for identity when they are part of the essence of an object. Arguably, a living thing, be it human, or any other animal, plant, insect etc. exists for a time continuously and, more or less, in a certain body. In other words, it is what it is for a certain time and in a certain spatially extended body[22]. And they essentially can't survive being unmade and put back together. Tables or statues also cannot survive annihilating changes, but in contrast to living things, can be dismembered and put back together.

Immaterial objects, such as the number 2 or prime number theory, are unchangeable because they don't have parts. Of course, in set theory, 2 can be defined based on the set that defines the number 1, and also the theory of prime numbers refers to other objects, specifically the prime numbers. But these are actually constituents of the real definitions of those objects. According to Fine, the constituents of the essence of an object are ontologically depended upon for the object in question to exist (Fine 1995, 275-276). In this sense, I consider intuitions regarding parts of immaterial objects to be more aptly intuitions about the "parents" of immaterial objects.

The actual world can be characterized as a non-rigid world, with one caveat. The distinction between essential and accidental properties is not necessarily a feature of objects in our world[23]. Material objects cease to exist, or at least cease to be causally relevant, if reduced to elementary particles that don't change, that only interact among each other. But in common language we heavily employ terms that refer to material objects, which we don't translate immediately into terms about

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[22] An endurantist perspective is assumed. I will not go into a perdurantist account of objects or worlds. Lewis (1986) discusses a perdurantist account of objects.

[23] And even if it is, the distinction isn't necessarily primitive. I see no reason to exclude other notions of essence from the analysis of a world with change. However, I suspect, that in the end, some other form of brute facts about essences must be relied upon to make an object be itself after change.

elemental particles[24]. This fact points to objects such as trees, cars or stars as actually existing in the world.

That A is a part of AB at one time and part of another object AC at a different time makes sense only if there is a "border" to separate the two objects. If such a border cannot clearly be discerned, then how can an object increase in parts and another decrease in parts?

5. Vague objects

Let's consider again the finite world with only two objects made up each of two parts: A_1A_2 (A) and B_1B_2 (B). Vagueness at the metaphysical level can be exemplified in this world by having both objects share at least one configuration: A (A_1A_2, A_1B_1, A_1B_2, $\mathbf{A_2B_1}$), and B (B_1B_2, $\mathbf{A_2B_1}$, A_2B_2). In the overlapping configuration (i.e. A_2B_1), A seems to have to doubled, since A_1B_2 (the remaining parts only configuration) is also A. Object B has disappeared, only to possibly reappear after a change to object A (the moving of parts from A to A).

Consider another case, that in which the two objects share all the mismatched configurations (A_1A_2, B_1B_2, A_1B_1, A_1B_2, A_2B_1, A_2B_2). Only when A is A_1A_2, and B is B_1B_2 the two objects are different. And because of the finite nature of the world, they are also synchronized in their identity. Necessarily when A is itself, B is itself.

The problem of vagueness in an infinite world isn't solved by appealing to essences in at least two cases. First, consider objects with not clearly defined "borders", like clouds[25] or puffs of smoke. The exact demarcation between water droplets inside the cloud and outside the cloud is not essential. Neither is defining a clear line between my right arm and the torso (or the rest of my body). For clouds and bodies, an approximation of their parts is sufficient most of the time.

Second, consider the case of simultaneous definitions: Sherlock Holmes and Doctor Watson. One object is essentially dependent on the

[24] It is debatable if such a translation is even possible. Where would we find a native speaker of elemental particles language to converse with and learn from?

[25] David Lewis discusses objects with vague borders in Lewis (1993).

other, and vice versa[26]. Similar with the finite world example, Sherlock Holmes cannot be identical with itself if Watson is not identical with itself. The two characters should be considered together in regard to their identity.

In both types of situations, vagueness can be better tackled at the level of language. Propositions about the exact border of a cloud seem meaningless. So do questions regarding the way in which Sherlock Holmes and Watson change. At the metaphysical level, objects would then be clearly defined (made of some parts as opposed to others), and language would be an imperfect way to express their identity after change.

Conclusion

The puzzles of change, when analyzed in the context of a collection of objects, a world, can be better understood and their implications can be resisted. I have sketched a holistic approach for increase and decrease considered together in a finite or infinite world.

To make objects be more than *ad hoc* configurations of parts, I rely on the notion of essence understood as primitive. The intuition is that some changes in parts lead to an object no longer being itself, e.g. removing sand for a heap of sand. While, after other changes in parts, the object remains the same, e.g. plucking a hair from the head of person. Objects have a certain lifetime in the world, characterized, in most cases, by space-time continuity.

The difference between changes that destroy the object and changes that do not is not always clear. I have gone over possible causes of vagueness at the metaphysical level, i.e. objects with no clearly defined borders and objects defined simultaneously.

I have also touched upon the similarity between the actual world and worlds (either finite or infinite) that permit change. Everyday beliefs and common language object-terms are a strong incentive to go further,

[26] Fine (1995, 282-283) discusses simultaneous definitions; objects are defined together, so as to not generate a cycle of ontological dependence.

and identify the actual world with one of the types of worlds that resists the paradoxes of change.

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NEW INDIRECT SYLLOGISTIC MOODS

GEORGETA CUCULEANU[1]

Abstract: *In this paper are presented the indirect syllogisms that are divided in two categories. These syllogisms are resulting from operation applied either to the premisses, being named primary indirect syllogisms, or to the conclusion of other syllogisms, being named secondary indirect syllogisms. For the first category three new primary indirect syllogisms have been established by us, by one in the second, third and fourth figures. For the second category thirteen indirect syllogisms are proposed. So, in the total, twenty-four indirect syllogisms are resulted, by six in every figure.*
Keywords: *indirect syllogism, mood, conversion, subalternation*

1. Introduction

Indirect syllogisms are those whose conclusion states the major term depending on the minor term, so that the major term is the subject and the minor term is the predicate. According to the way in which the conclusion is got, they are:

- primary or genuine indirect syllogisms, whose conclusion results from the operations applied only to the premisses;
- indirect syllogisms by conversion, the conclusion of which is obtained by conversion of the conclusion of the direct syllogisms;
- indirect syllogisms by subalternation, the conclusion of which is resulting by subalternation to the conclusion of indirect syllogisms with universal conclusion.

[1] PhD Candidate, University of Bucharest, Faculty of Philosophy. E-mail: cuculeanu@gmail.com

Indirect syllogisms by conversion and subalternation are secondary syllogisms, because they are obtained by operations applied to the conclusions of other syllogisms.

2. Establishing of new indirect syllogistic moods

The syllogisms of the first category result from certain combinations of premisses. Some combinations are indicated by Aristotle himself at the beginning of the chapter 7 of the first part of *Prior Analytics*:

> It is obvious [...] that, in all the figures, whenever there is no syllogism, if both terms are affirmative or negative, nothing is necessary; but if one is affirmative and the other negative, and if the negative is taken universally, there will always be a syllogism relating the minor term to the major: for example, if A belongs to all or some B, and B does not belong to any C, [...] it is necessary that C does not belong to some A. (*An. pr*., I, 7, 29a, 28-36).

Thus, "in all the figures", if the premisses are according to the text, primary indirect syllogisms will result. The premisses of these syllogisms are specified by the example given: the major premiss must be affirmative, universal or particular, and the minor premiss must be negative universal.

Another combination of premisses from which primary indirect syllogisms are resulting were established by us; it will be presented, in the paper, at the right place.

Following the indications given in the quoted text, Theophrastus (Dumitriu 1969, 189) established, for the first figure, two indirect syllogistic moods that were named by scholastic *Fapesmo*, the one with the universal-affirmative major premiss and the universal-negative minor premiss, and *Frisesomorum*, the one with the particular-affirmative major premiss and the universal-negative minor premiss (Didilescu and Botezatu 1976, 100); the conclusion of these moods is "Some P is not S".

For the second figure Iulius Pacius (1550-1635) established, also based on the quoted text, the primary indirect syllogistic mood *Firesmo*,

with the particular-affirmative major premiss and the universal-negative minor premiss and the conclusion "Some P is not S". The second primary indirect syllogism of this figure, with the universal-affirmative major premiss and the universal-negative minor premiss, is not recognized, because its conclusion resulting from the operations applied to these premisses and stating the major term depending on the minor term, was converted giving the direct syllogistic mood *Camestres*. The conversion of the conclusion was made by Aristotle himself, as it appears from the text:

> [...] if M [the middle] belongs to the whole N [the major], but not to O [the minor], then N will not belong to any O. For if M does not belong to any O, neither O belongs to any M; but M (as has been said) belongs to all N; then O will not belong to any N because the first figure has been made again. But since the negative relation is convertible, N will not belong to any O. (*An. pr*., I, 5, 27a, 11-17)

However, as Aristotle says, two chapters later, that "in all the figures" the syllogisms that fulfil the conditions mentioned in the first quoted text, report "the minor term to the major", he implicitly recognizes the existence of the primary indirect syllogism of the second figure with the conclusion "No P is S", which can be called *Cameste* and from which the mood *Camestres* is resulting by conversion of the conclusion. The idea of difference between the moods *Camestres* and *Cameste* also arises from Lukasiewicz's following text (1958, 27): "It is important that propositions of the type 'A belongs to no B' and 'B belongs to no A' are regarded by Aristotle as different".

Hence "All S is P" is different from "All P is S", what justifies the mood *Cameste*. Consequently, the following are highlighted: 1) the second figure also fulfils the conditions of the Aristotelian text; 2) the direct mood *Camestres* derives from an indirect mood by conversion of its conclusion.

The combination of premisses that leads to the primary indirect syllogisms established by us consists of a particular-negative premiss and a universal-affirmative one; it is applicable only to the second and third figures.

For the second figure, the primary indirect syllogism with this combination of premisses has the particular-negative major premiss and the universal-affirmative minor premiss:

$$\begin{array}{l} \text{Some P is not M} \\ \text{All S is M} \\ \hline \text{Some P is not S} \end{array} \qquad (1)$$

The proof of the conclusion is made by the indirect method of *reductio ad impossibile*; its contradictory is the sentence "All P is S" and replaces the major premisse in the mentioned syllogism, so that are obtained the premisses of the mood *Bramantip*:

$$\begin{array}{l} \text{All P is S} \\ \text{All S is M} \end{array} \qquad (2)$$

By transposing these premisses the mood *Barbara* is achieved:

$$\begin{array}{l} \text{All S is M} \\ \text{All P is S} \\ \hline \text{All P is M} \end{array} \qquad (3)$$

the conclusion of which is contradictory to the major premiss of the initial syllogism (1), so "Some P is not S" is the conclusion of the syllogism (1). The proposed name for the syllogism analyzed is *Brocamo*, where: *b* shows that the syllogism is reduced to the mood *Barbara*; group *br* specifies that it is obtained the mood *Bramantip*, as an intermediate syllogism; *c* indicates the replacement of the major premiss with the contradictory of the conclusion; *m* indicates the permutation of the minor premiss with the major one.

To illustrate the deductive process of *reductio ad impossibile* used to prove the conclusion of the syllogism (1), we propose the diagram in figure 1.

The symbols used have the following significations:

'A', 'O', are constants and have the known meaning;

'p' – major term, 'm' – middle term, 's' – minor term;

curly braces, '{}', delimit the contradictory of a sentence;

the double arrow, '⟹', shows the sense of the contradictory transformation;

the simple arrow, '→', indicates the sequence of deductive operations.

Tr indicates a transposition of premisses

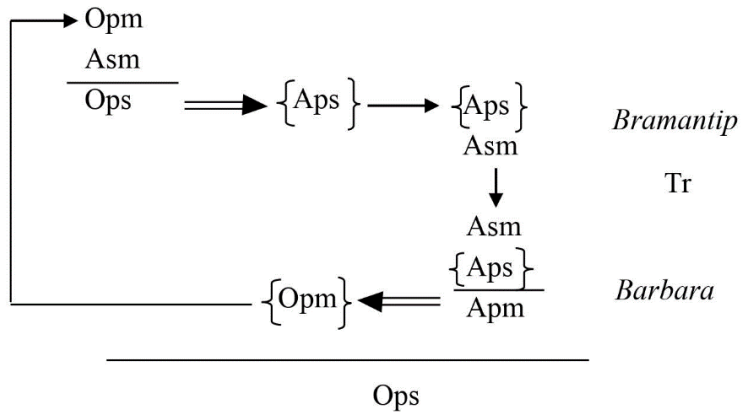

Figure 1

The graphical representation proposed for the deductive process of *reductio ad impossibile* is suggestive and has the following advantages over that used by Clark (1980, 9-11): 1) clarity of the demonstration operations and their sequence; 2) highlighting the initial syllogism as the starting and closing point of the deductive process; 3) vertical writing of the syllogism premisses facilitates the indication of the performed operations. The closing of the deductive process discharges the assumption that the conclusion of the initial syllogism is the contradictory to its conclusion and proves this conclusion. With this indirect syllogistic mood, the second syllogistic figure is enriched with another primary indirect syllogistic mood.

The deductive process represented in the figure 1 can be written in the natural deduction form. The symbols used, besides the precedents, are:

square brackets '[' , ']', for the numbers of premisses;

' ~ ' – sign of negation;

'Cs' – conclusion;

'Cd' – contradictory;

r.a.i. – *reductio ad impossibile.*

The rules of inference used are the moods *Bramantip* and *Barbara*.

For an exact preservation of the sequence of the deductive process operations represented diagrammatically, it is necessary to repeat some lines, what is indicated by the word "repetition".

[1] (1) Opm	Pr
[2] (2) Asm	Pr
[1,2] (3) Ops	1,2 Cs
~ [1,2] (4) Aps	3 Cd
[2] (5) Asm	Pr, repetition
	4,5 *Bramantip*
[2] (6) Asm	4,5 Tr
~ [1,2] (7) Aps	6,7 *Barbara*
~ [1,2] ∪ [2] (8) Apm	6,7 Cs
~ [1] (9) Apm	8 Cs
[1] (10) Opm	9 Cd
[1] (11) Opm	Pr, r.a.i.
[1,2] (12) Ops	1,2 Cs

In the first column on the left side are written only the premisses and their negation.

The deductive process can, also, be written using the rules of ~elimination and ~introduction of the Gentzen's system of natural deduction, because it corresponds to *reductio ad impossibile*. In this system the proof can be done both in the propositional logic and in the monadic predicate logic. In the propositional logic the proof is:

1 (1)	Opm	Pr
2 (2)	Asm	Pr
1,2 (3)	Ops	1,2 Cs
4 (4)	~ Ops	assumption
4 (5)	Aps	4 Cd
2,4 (6)	Apm	2,5 Cs *Barbara*
2,4 (7)	~ Opm	6 Cd
1,2,4 (8)	∧	1,7 ~E
1,2 (9)	~ ~ Ops	4,8 ~I
1,2 (10)	Ops	9 DN

This writing is more compact then first, but it does not illustrate all deductive process.

In monadic predicate logic the proof of the syllogism (1) is:

(∃x) (Px•~Mx), (∀x) (Sx→Mx) ⊢ (∃x) (Px•~Sx)

1 (1)	(∃x) (Px•~Mx)	Pr
2 (2)	(∀x) (Sx→Mx)	Pr
3 (3)	~(∃x) (Px•~Sx)	assumption
3 (4)	(∀x) ~ (Px•~Sx)	3 QS
3 (5)	(∀x) (Px→Sx)	4 DeM
3 (6)	Pa→Sa	5 ∀E
2 (7)	Sa→Ma	2 ∀E
8 (8)	Pa	assumption
3,8 (9)	Sa	6,8 → E (MP)
2,3,8 (10)	Ma	7,9 → E (MP)
2,3 (11)	Pa→Ma	8,10 → I
2,3 (12)	(∀x) (Px→Mx)	11 ∀I
2,3 (13)	~(∃x) ~(Px→Mx)	12 QS
2,3 (14)	~(∃x) (Px•~Mx)	13 DeM
1,2,3 (15)	∧	1,14 ~E
1,2 (16)	~ ~ (∃x) (Px•~Sx)	3,15 ~I
1,2 (17)	(∃x) (Px•~Sx)	16 DN

The Gentzen's system of monadic predicate logic allows, also, the use of direct method for proving the syllogisms (1) and (4). For the syllogism (1) the direct proof is:

$(\exists x)\,(Px \bullet \sim Mx), (\forall x)(Sx \rightarrow Mx) \vdash (\exists x)\,(Px \bullet \sim Sx)$

1	(1) $(\exists x)\,(Px \bullet \sim Mx)$	Pr
2	(2) $(\forall x)\,(Sx \rightarrow Mx)$	Pr
3	(3) $Pa \bullet \sim Ma$	assumption
2	(4) $Sa \rightarrow Ma$	2 $\forall$E
3	(5) $\sim Ma$	3 $\bullet$E
2,3	(6) $\sim Sa$	4,5 MT
3	(7) Pa	3 $\bullet$E
2,3	(8) $Pa \bullet \sim Sa$	6,7 $\bullet$I
2,3	(9) $(\exists x)\,(Px \bullet \sim Sx)$	8 $\exists$I
1,2	(10) $(\exists x)\,(Px \bullet \sim Sx)$	1,3,9 $\exists$E

For the third figure, Iulius Pacius has established, from the same indications of Aristotle, the primary indirect moods *Fapemo* and *Frisemo* (Didilescu and Botezatu 1976, 100); the conclusion of these syllogisms is "Some P is not S". In the case of this figure the combination of premisses established by us is composed of the universal-affirmative major premiss and the particular-negative minor premiss:

$$\begin{array}{l} \text{All M is P} \\ \underline{\text{Some M is not S}} \\ \text{Some P is not S} \end{array} \qquad (4)$$

and gives the third primary indirect syllogistic mood of this figure. The proof of the conclusion is made, also, by *reductio ad impossibile*. The contradictory of the accepted conclusion is "All P is S" and it will replace the minor premiss in the initial syllogism (4). Thus, it is obtained the mood *Bramantip*:

$$\begin{array}{l} \text{All M is P} \\ \text{All P is S} \end{array} \qquad (5)$$

Transposing the premisses the mood *Barbara* is got:

$$\frac{\begin{array}{l}\text{All P is S}\\ \text{All M is P}\end{array}}{\text{All M is S}} \qquad (6)$$

Its conclusion is the contradictory of the minor premiss of the initial indirect syllogism (4). Hence, the conclusion established for the initial syllogism (4) is correct. The proposed name for the syllogism (4) is *Bramoco*, where the letters have the same meaning as in the previous case. The graphical representation of the proof of the conclusion of this syllogism by *reductio ad impossibile* is given in figure 2.

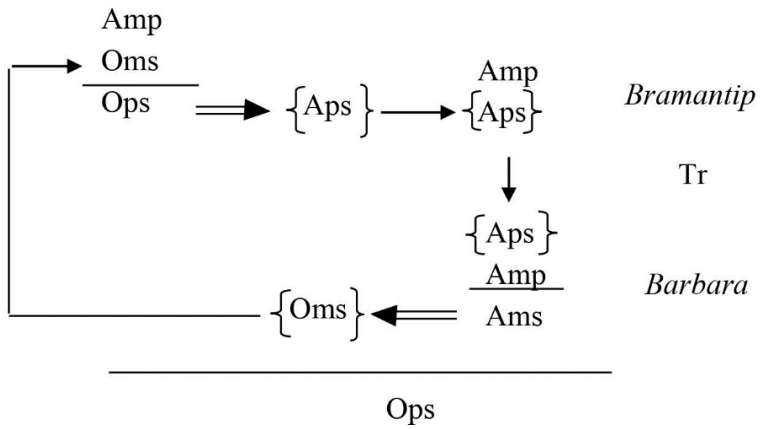

Figure 2

The natural deductive form of the representation of the figure 2 is:

[1] (1) Amp	Pr
[2] (2) Oms	Pr
[1,2] (3) Ops	1,2 Cs
~ [1,2] (4) Aps	3 Cd
[1] (5) Amp	Pr, repetition
~ [1,2] (6) Aps	3 Cd, repetition
	5,6 *Bramantip*

~ [1,2] (7) Aps	5,6 Tr
[1] (8) Amp	7,8 *Barbara*
~ [1,2] ∪ [1] (9) Ams	7,8 Cs
~ [2] (10) Ams	9 Cs
[2] (11) Oms	10 Cd
[2] (12) Oms	Pr., r.a.i.
[1,2] (13) Ops	1,2 Cs

Like above, the deductive process of the figure 2 can be written in the Gentzen's, system of natural deduction for propositional logic too.

1 (1) Amp	Pr
2 (2) Oms	Pr
1,2 (3) Ops	1,2 Cs
4 (4) ~ Ops	assumption
4 (5) Aps	4 Cd
1,4 (6) Ams	5,1 Cs *Barbara*
1,4 (7) ~ Oms	6 Cd
1,2,4 (8) ∧	2,7 ~E
1,2 (9) ~ ~ Ops	4,8 ~I
1,2 (10) Ops	9 DN

The proof in the monadic predicate logic of the syllogism (4) is:

(∀x) (Mx→Px), (∃x) (Mx•~Sx) ⊢ (∃x) (Px•~Sx)

1 (1) (∀x) (Mx→Px)	Pr
2 (2) (∃x) (Mx•~Sx)	Pr
3 (3) ~(∃x) (Px•~Sx)	assumption
3 (4) (∀x) ~(Px•~Sx)	3 QS
3 (5) (∀x) (Px→Sx)	4 DeM
3 (6) Pa→Sa	5 ∀E
1 (7) Ma→Pa	1 ∀E
8 (8) Ma	assumption
1,8 (9) Pa	7,8 → E (MP)
1,3,8 (10) Sa	6,9 → E (MP)
1,3 (11) Ma→Sa	8,10 → I

1,3 (12)	(∀x) (Mx→Sx)	11 ∀I
1,3 (13)	~(∃x) ~(Mx→Sx)	12 QS
1,3 (14)	~(∃x) (Mx•~Sx)	13 DeM
1,2,3 (15)	∧	2,14 ~E
1,2 (16)	~ ~ (∃x) (Px•~Sx)	3,15 ~I
1,2 (17)	(∃x) (Px•~Sx)	16 DN

The direct proof of the syllogism (4), using Gentzen's system, is:

(∀x) (Mx→Px), (∃x) (Mx•~Sx) ⊢ (∃x) (Px•~Sx)

1 (1)	(∀x) (Mx→Px)	Pr
2 (2)	(∃x) (Mx•~Sx)	Pr
1 (3)	Ma→Pa	1 ∀E
4 (4)	Ma•~Sa	assumption
4 (5)	Ma	4 •E
1,4 (6)	Pa	3,5 → E (MP)
4 (7)	~Sa	4 •E
1,4 (8)	Pa•~Sa	6,7 •I
1,4 (9)	(∃x) (Px•~Sx)	8 ∃I
1,2 (10)	(∃x) (Px•~Sx)	2,4,9 ∃E

Another primary indirect syllogistic mood of the third figure can be considered the one whose conclusion "Some P is S" was converted for obtaining the direct mood *Disamis*. This syllogism is:

Some M is P
All M is S
———————
Some P is S (7)

As a name for this syllogism is proposed *Disami*, which highlights the fact that the mood *Disamis* was got by conversion of the conclusion of the syllogism (7). With this syllogism, the third figure has four primary indirect syllogisms.

Between the new-established primary indirect moods, *Brocamo*, for the second figure, and *Bramoco*, for the third figure, and the direct

moods *Baroco* and *Bocardo* of the two figures, there is an analogy given by the following characteristics:

a) all have a particular-negative premiss and the other universal-affirmative, a combination of premisses which can only be found in the second and third figures, where the middle term has the same function in both premisses;

b) for the same figure the major premiss of the direct mood becomes the minor premiss in the indirect mood and vice versa, the minor premiss of the direct mood becomes the major premiss in the indirect mood;

c) the direct mood of the second figure becomes indirect mood in the third figure, and the direct mood of the third figure becomes indirect mood in the second figure, respecting the position of the middle term of each figure;

d) the conclusion, both of the direct and the indirect moods, is proved by *reductio ad impossibile* which reduces them all to the *Barbara* mood. The indirect moods are reduced to *Barbara* by means of the mood *Bramantip*.

For the fourth figure, we have established the following primary indirect syllogism, which fulfils the conditions given by Aristotle:

$$\begin{array}{c} \text{Some P is M} \\ \text{No M is S} \\ \hline \text{Some P is not S} \end{array} \qquad (8)$$

By transposing the premisses one obtains the mood *Ferio*:

$$\begin{array}{c} \text{No M is S} \\ \text{Some P is M} \\ \hline \text{Some P is not S} \end{array} \qquad (9)$$

As a name for the syllogism (8) it is proposed *Fimeno*, where the letters *i*, *e*, *m* and *o* have the known significations and *n* helps to pronounce the word.

The syllogism (8) can be, also, proved by Gentzen's system of natural deduction, in the monadic predicate logic, but using only the direct method.

$(\exists x)(Px \bullet Mx), (\forall x)(Mx \rightarrow \sim Sx) \vdash (\exists x)(Px \bullet \sim Sx)$

1	(1) $(\exists x)(Px \bullet Mx)$	Pr
2	(2) $(\forall x)(Mx \rightarrow \sim Sx)$	Pr
3	(3) $Pa \bullet Ma$	assumption
2	(4) $Ma \rightarrow \sim Sa$	2 ∀E
3	(5) Ma	3 •E
3	(6) Pa	3 •E
2,3	(7) $\sim Sa$	4,5 MT
2,3	(8) $Pa \bullet \sim Sa$	6,7 •I
2,3	(9) $(\exists x)(Px \bullet \sim Sx)$	8 ∃I
1,2	(10) $(\exists x)(Px \bullet \sim Sx)$	1,3,9 ∃E

The second indirect syllogistic mood specified in Aristotle's text – with the universal-affirmative major premiss and the universal-negative minor premiss was used, as in the case of the second figure, for obtaining the direct mood *Camenes*. Since the conclusion of this mood is the converse of the conclusion that states the major term depending on the minor term resulting from the mood *Celarent* to which the syllogism with the mentioned premisses is reduced, the syllogism:

$$\begin{array}{l} \text{All P is M} \\ \text{No M is S} \\ \hline \text{No P is S} \end{array} \qquad (10)$$

can be considered as a primary indirect mood of the fourth figure for which one proposes as name *Camente*. By these two primary indirect syllogistic moods, the Aristotelian expression "in all the figures" also includes the fourth figure. A similar discussion can be made about the moods *Bramantip* and *Dimaris*; each of them results from a primary indirect mood whose names can be *Bramana*, with the same premisses like *Bramantip* and the conclusion "All P is S", respectively *Dimari*, with the same premisses like *Dimaris* and the conclusion "Some P is S".

3. Proposing new indirect syllogistic moods

Conversion and subalternation of the conclusion of certain syllogisms, as methods for getting other syllogisms, are specified by Aristotle himself in the text:

> Because some syllogisms are universal and others particular, all universal syllogisms give more than one conclusion, and of the particular ones, the affirmative ones give more than a conclusion, while the negative ones give only the usual conclusion. (*An. pr.*, II, 1, 53a, 3-8).

According to the text, the universal syllogisms give two further conclusions, one by the conversion of the "usual conclusion" and the other by the subalternation of the "usual conclusion". The particular-affirmative syllogisms give only one conclusion obtained by conversion of "usual conclusion". By "usual conclusion" Aristotle means the conclusion that states the minor term depending on the major term, even if it is obtained by conversion of the conclusion resulting from the operations applied to the given premisses, as is the case of the mood *Camestres*.

The text just quoted is applied to all direct and indirect syllogisms. In this paper, will be discussed only the getting of the indirect syllogisms by conversion and subalternation. In the first figure, the indirect moods *Baralipton*, *Celantes* and *Dabitis* were obtained by conversion of the conclusions of the direct moods *Barbara*, *Celarent* and *Darii*, respectively (Didilescu and Botezatu 1976, 101). The conclusions of the three indirect moods are: "Some P is S", "No P is S" and "Some P is S", respectively. These moods were established by Theophrastus (Dumitriu 1969, 186).

If one follows the example of the direct moods of subalternation of their conclusion, a method by which a total of five direct subaltern moods have been obtained, other indirect syllogistic moods can be obtained in all the figures. For the first figure, as indirect subaltern mood can be considered *Celanto*, from *Celantes*, with the conclusion "Some P is not S".

In the second figure, can be obtained by conversion the indirect mood *Cesares* with the conclusion "No P is S" resulting from the conclusion of the direct mood *Cesare*. By subalternation of the conclusion

of the indirect moods *Cesares* and *Cameste*, will be getting other two indirect moods with the same conclusion "Some P is not S" and for which we propose the names *Cesareso* (to distinguish it from the direct subaltern mood *Cesaro*), respectively *Camesto*.

In the third figure two indirect moods can be obtained, both with the particular-affirmative conclusion "Some P is S" resulting by conversion of the particular-affirmative conclusion "Some S is P" of the direct moods *Darapti* and *Datisi*. As names are proposed *Daraptis* and *Datisis*. Indirect moods by subalternation cannot be obtained in this figure.

In the fourth figure, indirect syllogistic moods cannot be obtained by conversion, because three of the direct moods, as it was shown above, were obtained by conversion of primary indirect syllogisms. Indirect syllogistic moods by subalternation result from the proposed primary indirect moods *Bramana* and *Camente*. For these indirect moods, the names proposed are *Bramanip* and *Camento*; the first one has the conclusion "Some P is S" and the second one has the conclusion "Some P is not S".

4. Conclusions

These which are presented above can be summarized as follows:

I. Primary indirect syllogistic moods are 13 of which:
   a) *new-established*, 3 – one in the second figure (*Brocamo*), one in the third figure (*Bramoco*) and one in the fourth figure (*Fimeno*);
   b) *recognized*, 5 – two in the first figure (*Fapesmo* and *Frisesomorum*), one in the second figure (*Firesmo*) and two in the third figure (*Fapemo* and *Frisemo*);
   c) *proposed*, 5 – one in the second figure (*Cameste*), one in the third figure (*Disami*) and three in the fourth figure (*Bramana*, *Camente*, *Dimari*).

II. Indirect syllogistic moods by conversion are 6 of which:
   a) *recognized*, 3 – in the first figure (*Baralipton*, *Celantes*, *Dabitis*);
   b) *proposed*, 3 – one in the second figure (*Cesares*) and two in the third figure (*Daraptis*, *Datisis*).

III. Indirect syllogistic moods by subalternation are 5, all *proposed*: one in the first figure (*Celanto*), two in the second figure (*Cesareso, Camesto*) and two in the fourth figure (*Bramanip, Camento*).

Consequently, there are 24 possible indirect syllogistic moods of which only 8 are recognized. By admitting the 3 new-established indirect moods and the 13 proposed ones, each syllogistic figure will have 6 indirect moods, what shows a numerical equivalence between the direct and indirect moods.

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