DETERMINING THE RELATIONSHIPS BETWEEN PREDICATION JUDGMENTS USING THEORY OF THE RELATIONS OF FLOREA ȚUȚUGAN

GEORGETA CUCULEANU[1]

Abstract. The paper presents the relationships between the predication judgments, starting from the unique and well-determinate relations that can exist between the two terms of a judgment. These relations are of identity, contradiction, subordination, contrariety, subcontrariety, superordination and crossing. For determining these relationships, the disjunctions of unique and well-determinate relations that represent every universal or particular judgment were established. It emerges from the study that universal judgments are represented by disjunctions of two unique relations, being double indeterminate. On the other hand, particular judgments are represented by disjunctions of five unique relations, thus being quintuple indeterminate. Relationships between judgments were established by comparing disjunctions.

Keywords: *judgments, opposition, relationships, terms, disjunction.*

I. Introduction

Relationships between predication judgments are relations between their statements and are given by relations between the two component terms of a judgment which may be positive and negative. The relations between the terms of a judgment and their graphical representation are given by Florea Țuțugan in his monumental book "*Silogistica judecăților de predicație*". In this book he proves that the formal structure of classical logic can be

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extended by introducing in the judgments both positive terms and negative terms. This allows the multiplication of syllogistic moods.

As method for determining these relations, in the paper is used the graphical representation of the relations that exist between the respective terms. This representation ensures the possibility to establish all the judgments that can be stated with two terms within each relation. Also, the method allows to establish the disjunctions of relations for every judgment and, making use of these disjunctions, the relationships between the predication judgments will be determined. Florea Țuțugan in his book gave disjunctions of relations only for fundamental predication judgment without justifying their origin. Also, as method for establishing the other judgments that can be obtained with two terms, positive and negative, he used conversion and obversion of the fundamental judgments.

In the present paper the results obtained by the method used in it, are compared with the ones given by Florea Țuțugan in his book for pointing out the equivalence of the two methods.

II. Brief presentation of Florea Țuțugan's theory

Florea Țuțugan begins his study, carried out in the spirit of classical logic, by representing the seven "unique and well-determinate" relations (*op. cit.* p. 7) discovered between two terms that are elements of the judgment "S is P", called predication judgment. These relations are, according to Florea Țuțugan, "simple and irreducible" (*Ibidem* p. 8) because they cannot be stated by means of others. In the judgment "S is P" copula, the terms or the whole judgment may be affirmative or negative. Also he states that relations between the two terms can be characterized, using subsumption and implication taking into account the extensions of the terms. The judgments of subsumption are judgments whose subject is a class of individuals or cases. The subject of implicative judgments is not class of individuals or cases (*op. cit.,* p. 14), so they have not determinative. Therefore, we can not talk about particular judgments in the case of the implicative judgments.

The shift from a subsumption judgment to an implication one is made easily using the following relations (*Ibidem,* pp. 15-19):

"All A is B" corresponds to "A includes B"
"No A is B" corresponds to "A excludes B"

For particular judgments, it is taken into account the fact that they are the negations of the universals of opposite quality. So:

"Some A is B" corresponds to "A does not exclude B"
"Some A is not B" corresponds to "A does not include B"

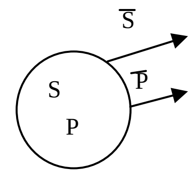
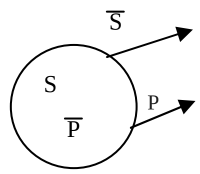
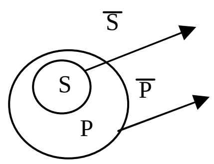
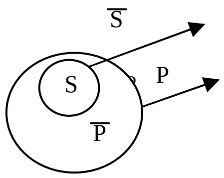
The seven relations were divided by Florea Țuțugan into three categories: category I, consisting of two relations (identity and contradiction) represented graphically in two equivalent ways; category II, consisting of four relations (subordination, contrariety, subcontrariety and superordination) each represented graphically in three equivalent ways; category III, consisting of one relation (crossing) represented graphically in four equivalent ways (*Ibidem* p. 9).

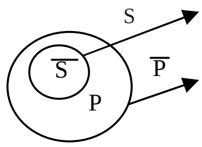
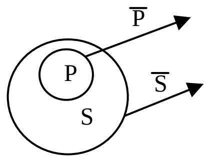
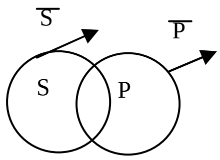
Classical logic did not take into account the relations of contradiction and subcontrariety, it is stated in the cited paper (p. 10) because classical logic was focused exclusively on positive terms. Also, by admitting the two relations between two terms of a judgment represents an extension of traditional logic, because they allow the use of negative terms. In this way, one obtains, in addition, other four judgments (A', E', I', O'), about which Florea Țuțugan states that "they are (...) perfect analogous to the classical judgments, with the only observation that they always have the negative subject" (*Ibidem* p. 12). It should be noted that the predicate of the four judgments is also negative.

III. Predication judgments for every relation

The seven relations, taken separately, are perfectly and totally determinate, and the disjunction of all constitutes is a totally indeterminate relation between the two terms (*Idem*). Combinations of two, three, to six irreducible relations are indeterminate (double, triple etc.). These unique and well-determinate relations that may exist between two positive and negative terms, each accompanied by the first graphical representation given by Florea Țuțugan in his book are presented in Table 1. Using the graphical representation, for each relation, all possible subsumption judgments have been established. It should be take into consideration the fact that a universal judgment involves a particular judgment of the same quality. The symbols of the relations in the table are the same as those used by Florea Țuțugan.

Table 1

The symbol and name of the relation	Relation scheme	Possible judgments	
I_1 – identity (equivalence, symmetric implication)		All S is P	All P is S
		Some S is P	Some P is S
		All $\overline{S}$ is $\overline{P}$	All $\overline{P}$ is $\overline{S}$
		Some $\overline{S}$ is $\overline{P}$	Some $\overline{P}$ is $\overline{S}$
		No S is $\overline{P}$	No $\overline{P}$ is S
		Some S is not $\overline{P}$	Some $\overline{P}$ is not S
		No $\overline{S}$ is P	No P is $\overline{S}$
		Some $\overline{S}$ is not P	Some P is not $\overline{S}$
I_2 – contradiction (exclusion, non-equivalence)		All S is $\overline{P}$	All $\overline{P}$ is S
		Some S is $\overline{P}$	Some $\overline{P}$ is S
		All $\overline{S}$ is P	All P is $\overline{S}$
		Some $\overline{S}$ is P	Some P is $\overline{S}$
		No S is P	No P is S
		Some S is not P	Some P is not S
		No $\overline{S}$ is $\overline{P}$	No $\overline{P}$ is $\overline{S}$
		Some $\overline{S}$ is not $\overline{P}$	Some $\overline{P}$ is not $\overline{S}$
II_1 – subordination (direct implication)		All S is P	All $\overline{P}$ is $\overline{S}$
		Some S is P	Some $\overline{P}$ is $\overline{S}$
		Some $\overline{S}$ is P	Some P is S
		Some $\overline{S}$ is $\overline{P}$	Some P is $\overline{S}$
		No S is $\overline{P}$	No $\overline{P}$ is S
		Some S is not $\overline{P}$	Some $\overline{P}$ is not S
		Some $\overline{S}$ is not P	Some P is not $\overline{S}$
		Some $\overline{S}$ is not $\overline{P}$	Some P is not S
II_2 – contrariety (strict positive exclusion)		All S is $\overline{P}$	All P is $\overline{S}$
		Some S is $\overline{P}$	Some P is $\overline{S}$
		No S is P	No P is S
		Some S is not P	Some P is not S
		Some $\overline{S}$ is $\overline{P}$	Some $\overline{P}$ is S
		Some $\overline{S}$ is not $\overline{P}$	Some $\overline{P}$ is not $\overline{S}$
		Some $\overline{S}$ is not P	Some $\overline{P}$ is $\overline{S}$
		Some $\overline{S}$ is P	Some $\overline{P}$ is not S

<table>
<tr><th>The symbol and name of the relation</th><th>Relation scheme</th><th>Possible judgments</th></tr>
<tr><td>II_3 – subcontrariety (strict negative exclusion)</td><td></td><td>All $\overline{S}$ is P All $\overline{P}$ is S
Some $\overline{S}$ is P Some $\overline{P}$ is S
No $\overline{S}$ is $\overline{P}$ No $\overline{P}$ is $\overline{S}$
Some $\overline{S}$ is not $\overline{P}$ Some $\overline{P}$ is not $\overline{S}$
Some S is P Some P is S
Some S is $\overline{P}$ Some P is $\overline{S}$
Some S is not P Some P is not $\overline{S}$
Some S is not $\overline{P}$ Some P is not S</td></tr>
<tr><td>II_4 – superordination (reverse implication)</td><td></td><td>All $\overline{S}$ is $\overline{P}$ All P is S
Some $\overline{S}$ is $\overline{P}$ Some P is S
No $\overline{S}$ is P No P is $\overline{S}$
Some $\overline{S}$ is not P Some P is not $\overline{S}$
Some S is P Some $\overline{P}$ is S
Some S is $\overline{P}$ Some $\overline{P}$ is $\overline{S}$
Some S is not P Some $\overline{P}$ is not S
Some S is not $\overline{P}$ Some $\overline{P}$ is not $\overline{S}$</td></tr>
<tr><td>III – crossing (implicative indifference)</td><td></td><td>Some S is P Some P is S
Some S is not P Some P is not S
Some S is $\overline{P}$ Some $\overline{P}$ is S
Some S is not $\overline{P}$ Some $\overline{P}$ is not S
Some $\overline{S}$ is P Some P is $\overline{S}$
Some $\overline{S}$ is not P Some P is not $\overline{S}$
Some $\overline{S}$ is $\overline{P}$ Some $\overline{P}$ is $\overline{S}$
Some $\overline{S}$ is not $\overline{P}$ Some $\overline{P}$ is not $\overline{S}$</td></tr>
</table>

From the analysis of the Table 1 result:

1. Every type of relation generates sixteen judgments, so that there are 112 judgments in total, of which 32 are universal judgments and 80 are particular judgments; some of them are obtained multiple times.
2. The sixteen judgments of each relation are divided by topic into two groups of eight judgments with the same subject, of which four with positive subject and four with negative subject. Also, the predicates are four positive and four negative. In each group there are, also, judgments with both terms of the same sign. Between the judgments of the two groups of a relation there is asymmetry regarding the function of the terms and symmetry regarding the quality of the terms and judgments;
3. Some judgments (universal or particular, affirmative or negative) characterize more relations;
4. There is no judgment that characterizes all relations.

IV. Disjunctions of relations of the judgments

By ordering the judgments in Table 1 by subject, quantity and quality, we end up with groups of four judgments with the same subject, the same quantity and quality, presented in Table 2. This table highlights the unique and well-determinate relations that characterize each of the judgments. The disjunctions of these relations are the same as those given by Florea Țuțugan, for the eight types of fundamental judgments (*Ibidem* pp. 10-12); he gave them without explanations regarding their obtaining.

Table 2

Judgment	Symbol	Disjunction of the relations	Judgment	Symbol	Disjunction of the relations
A All S is P	SaP	$I_1 \vee II_1$	All P is S	PaS	$I_1 \vee II_4$
All S is $\bar{P}$	$Sa\bar{P}$	$I_2 \vee II_2$	All P is $\bar{S}$	$Pa\bar{S}$	$I_2 \vee II_2$
All $\bar{S}$ is P	$\bar{S}aP$	$I_2 \vee II_3$	All $\bar{P}$ is S	$\bar{P}aS$	$I_2 \vee II_3$
A' All $\bar{S}$ is $\bar{P}$	$\bar{S}a\bar{P}$	$I_1 \vee II_4$	All $\bar{P}$ is $\bar{S}$	$\bar{P}a\bar{S}$	$I_1 \vee II_1$
E No S is P	SeP	$I_2 \vee II_2$	No P is S	PeS	$I_2 \vee II_2$
No S is $\bar{P}$	$Se\bar{P}$	$I_1 \vee II_1$	No P is $\bar{S}$	$Pe\bar{S}$	$I_1 \vee II_4$
No $\bar{S}$ is P	$\bar{S}eP$	$I_1 \vee II_4$	No $\bar{P}$ is S	$\bar{P}eS$	$I_1 \vee II_1$
E' No $\bar{S}$ is $\bar{P}$	$\bar{S}e\bar{P}$	$I_2 \vee II_3$	No $\bar{P}$ is $\bar{S}$	$\bar{P}e\bar{S}$	$I_2 \vee II_3$
I Some S is P	SiP	$I_1 \vee II_1 \vee II_3 \vee II_4 \vee III$	Some P is S	PiS	$I_1 \vee II_1 \vee II_3 \vee II_4 \vee III$
Some S is $\bar{P}$	$Si\bar{P}$	$I_2 \vee II_2 \vee II_3 \vee II_4 \vee III$	Some P is $\bar{S}$	$Pi\bar{S}$	$I_2 \vee II_1 \vee II_2 \vee II_3 \vee III$
Some $\bar{S}$ is P	$\bar{S}iP$	$I_2 \vee II_1 \vee II_2 \vee II_3 \vee III$	Some $\bar{P}$ is S	$\bar{P}iS$	$I_2 \vee II_2 \vee II_3 \vee II_4 \vee III$
I' Some $\bar{S}$ is $\bar{P}$	$\bar{S}i\bar{P}$	$I_1 \vee II_1 \vee II_2 \vee II_4 \vee III$	Some $\bar{P}$ is $\bar{S}$	$\bar{P}i\bar{S}$	$I_1 \vee II_1 \vee II_2 \vee II_4 \vee III$
O Some S is not P	SoP	$I_2 \vee II_2 \vee II_3 \vee II_4 \vee III$	Some P is not S	PoS	$I_2 \vee II_1 \vee II_2 \vee II_3 \vee III$
Some S is not $\bar{P}$	$So\bar{P}$	$I_1 \vee II_1 \vee II_3 \vee II_4 \vee III$	Some P is not $\bar{S}$	$Po\bar{S}$	$I_1 \vee II_1 \vee II_3 \vee II_4 \vee III$
Some $\bar{S}$ is not P	$\bar{S}oP$	$I_1 \vee II_1 \vee II_2 \vee II_4 \vee III$	Some $\bar{P}$ is not S	$\bar{P}oS$	$I_1 \vee II_1 \vee II_2 \vee II_4 \vee III$
O' Some $\bar{S}$ is not $\bar{P}$	$\bar{S}o\bar{P}$	$I_2 \vee II_1 \vee II_2 \vee II_3 \vee III$	Some $\bar{P}$ is not $\bar{S}$	$\bar{P}o\bar{S}$	$I_2 \vee II_2 \vee II_3 \vee II_4 \vee III$

The table shows the internal logical structure of judgments and confirms the author's assertion that universal judgments are double indeterminate, being the disjunction of two unique and well-determine relations, and the particular judgments are quintuple indeterminate, being disjunction of five unique and well-determinate relations (*Ibidem* p. 11). It is noted that two terms, positive and negative, lead exactly to 32 possible distinct and simple judgments, which is in line with the statements of Florea Țuțugan contained in pages 13-15 of the above-mentioned book, judgments which were written by him without emphasizing their connection with the graphical representations of the seven unique and well-determinate relations. From the 32 judgments, sixteen are universal and sixteen are particular. As for the subject, sixteen have the subject S and sixteen have the subject P. At the same time, Florea Țuțugan states that these judgments are the only possible disjunctions of the seven irreducible relations enunciated by a simple judgment of the form "S-P" (*Ibidem* p. 12), judgments that can be expressed one by another using negation.

His assertion that "affirmative judgments necessarily comprise the relation I_1 and do not comprise the I_2 relation" (*Idem*) certainly refers only to the A, A', I and I' judgments, since Table 2 shows that affirmative judgments having a single negative term in their composition do not contain in their disjunction the relation I_1 but contain I_2. The author makes a similar observation about the negative judgments as they all include the relation I_2, and they do not include the relation I_1 (*Idem*). It is certain that Florea Țuțugan refers to judgments E, E', O and O', since the Table 2 shows that all negative judgments that contain only one negative term have in their disjunction the relation I1 and do not have the relation I_2.

It can be checked whether Table 2 comprises all the judgments from the Table 1 using the number of distinct universal and particular judgments and the number of relations that make up a disjunction. The calculation results in 112 judgments, equal to the number of judgments from the Table 1.

Analyzing the disjunctions of unique and well-determinate relations characterizing the judgments in Table 2, we can see the following:

1. All universal judgment groups, irrespective of their quality and subject matter, contain the same four different disjunctions of two unique and well-determinate relations, one of the first and one of the second categories. Each disjunction characterizes only a judgment from every group of universal judgments. This means that there is a correspondence between these judgments, correspondence that will be discussed further;
2. All groups of particular judgments, irrespective of their quality and subject matter, are characterized by the same four different

disjunctions of five unique and well-determinate relations, indicating that there is a certain correspondence between these judgments;

3. Relations I_1 (identity), II_1 (subordination) and II_4 (superordination) are components of the disjunctions of universal and particular judgments, which contain no negations or contain an even number of negations;
4. Relations I_2 (contradiction), II_2 (contrariety) and II_3 (subcontrariety) are components of disjunctions that characterize universal and particular judgments that have an odd number of negations;
5. Relation III (crossing) is part of the disjunctions that characterize all and only particular judgments;
6. The disjunctions of the judgments confirm the Aristotelian theory of the conversion of judgments (Aristotel, *An. pr.* I, 2). So:
   a) the disjunctions corresponding to affirmative universal judgments are part of the disjunctions that characterize particular affirmative judgments with the same terms, but in reverse order. As a result, an affirmative universal judgment with either positive or negative terms has as converse a particular affirmative judgment with inverted terms, although the two judgments (universal and particular) are not equivalent. In order for an affirmative universal judgment (All A is B) to have as converse an affirmative universal judgment (All B is A), it is necessary that the only relation existing between its two terms to be of identity, which requires that their extensions to be equal. This requirement is also highlighted by the disjunctions of the two judgments in which the first component is the same (I_1). However, Table 2 shows that for each affirmative universal judgment there is a judgment of the same quantity and quality, but with inverse negative terms, characterized by the same disjunction;
   b) the pairs of negative universal judgments with the same terms, but inverted to each other are characterized by the same disjunction, so they are converse one to other; the same characteristic is also given by the pairs of affirmative

particular judgments, which proves that one is the converse of the other;

c) the pairs of particular negative judgments characterized by the same disjunction have opposing and reversed terms. As a result, the Aristotelian theory of conversion is confirmed, established only for positive terms, as that negative particular judgment has no converse.

If conversion is only meant to preserve the quantity and quality of judgment and the reversal of terms, Table 2 shows that both the universal affirmative judgments and the negative one "can be converted", but by changing the sign of the terms. For example, the judgment "All S is P" is "converse" to the judgment "All $\overline{P}$ is $\overline{S}$", and the judgment "Some S is not P" is "converse" to the judgment "Some $\overline{P}$ is not $\overline{S}$", because the judgments of each pair are equivalent, being characterized by the same disjunction. These are also stated by Florea Țuțugan in his book (*Ibidem* pp. 55-56). In order to decide whether the judgments that make up each of the two pairs can be considered as "converses" one to another, the example method can be used. In the case of affirmative universal judgments, one considers the syllogism of *Cesare* mood:

No dielectric is conductive.
<u>All metals are conductive.</u>
No metal is dielectric.

Applying to it the operations: 1) replacing the minor premise with its equivalent with negative terms "All non-conductors are non-metals; 2) obversion of the major premise: "All dielectrics are non-conductive"; 3) the transposition of the new premises, one obtains the *Barbara* syllogism:

All non-conductors are non-metals.
<u>All dielectrics are non-conductive.</u>
All dielectrics are non-metals.

By obversion of the conclusion and the conversion of the obverse results the conclusion of the given syllogism. Therefore, it can be considered that an affirmative universal judgment with positive terms has as "converse" a universal affirmative judgment with negative and reversed terms.

In the case of negative particular judgments, one considers the syllogism *Bocardo*:

Some artworks are not paintings.
<u>All artworks are artistic products.</u>
Some artistic products are not paintings.

By replacing the premises with their equivalents with negative terms: "Some non-paintings are not non-artworks" and "All non-artistic products are non-artworks" a *Baroco* syllogism is obtained, with the conclusion "Some non-paintings are not non-artistic products" equivalent to "Some artistic products are not paintings". So it can be considered that the negative particular judgment "Some S is not P" has as "converse" the negative particular judgment "Some $\overline{P}$ is not $\overline{S}$".

From the two examples it can be concluded that the use of negative terms, together with the positive ones, leads to the expansion of the conversion.

7. The judgments of Table 2 also check the theory of immediate inferences, as can easily be seen from the comparison of the data in this table with those in Table 3, drawn up after the explanations from Didilescu (*op. cit*. pp. 48-56) for judgments A, E, I and O and completed by us with the judgments A', E', I' and O''.

Table 3

Initial judgment	Conversion	Obversion	Contraposition		Inversion	
			partial	partial	partial	total
A SaP	PiS	$Se\bar{P}$	$\bar{P}eS$	$\bar{P}a\bar{S}$	$\bar{S}oP$	$\bar{S}i\bar{P}$
E SeP	PeS	$Sa\bar{P}$	$\bar{P}iS$	$\bar{P}o\bar{S}$	$\bar{S}iP$	$\bar{S}o\bar{P}$
I SiP	PiS	$So\bar{P}$	-	-	-	-
O SoP	-	$Si\bar{P}$	$\bar{P}iS$	$\bar{P}o\bar{S}$	-	-
A' $\bar{S}a\bar{P}$	$\bar{P}i\bar{S}$	$\bar{S}eP$	$Pe\bar{S}$	PaS	$So\bar{P}$	SiP
E' $\bar{S}e\bar{P}$	$\bar{P}e\bar{S}$	$\bar{S}aP$	$Pi\bar{S}$	PoS	$Si\bar{P}$	SoP
I' $\bar{S}i\bar{P}$	$\bar{P}i\bar{S}$	$\bar{S}oP$	-	-	-	-
O' $\bar{S}o\bar{P}$	-	$\bar{S}iP$	$Pi\bar{S}$	PoS	-	-

V. Relationships between predication judgments

Relationships between judgments are similar to the unique and well-determinate relations that exist between the terms of a judgment. The definitions of these relationships given by Florea Țuțugan in his book (p. 21) are also used in this paper. For highlighting these relationships, the judgments in Table 2 were ordered after their disjunctions, resulting the Table 4.

Table 4

<table>
<tr><th colspan="4">Universal judgments</th></tr>
<tr><th colspan="4">Disjunction of the relations</th></tr>
<tr><th>$I_1 v II_1$</th><th>$I_2 v II_2$</th><th>$I_1 v II_4$</th><th>$I_2 v II_3$</th></tr>
<tr><td>A All S is P
No S is $\bar{P}$
No $\bar{P}$ is S
All $\bar{P}$ is $\bar{S}$</td><td>E No S is P
All S is $\bar{P}$
No P is S
All P is $\bar{S}$</td><td>A' All $\bar{S}$ is $\bar{P}$
No $\bar{S}$ is P
No P is $\bar{S}$
All P is S</td><td>E' No $\bar{S}$ is $\bar{P}$
All $\bar{S}$ is P
All $\bar{P}$ is S
No $\bar{P}$ is $\bar{S}$</td></tr>
<tr><th colspan="4">Particular judgments</th></tr>
<tr><th colspan="4">Disjunction of the relations</th></tr>
<tr><th>$I_1 v II_1 v II_3 v II_4 v III$</th><th>$I_2 v II_2 v II_3 v II_4 v III$</th><th>$I_1 v II_1 v II_2 v II_4 v III$</th><th>$I_2 v II_1 v II_2 v II_3 v III$</th></tr>
<tr><td>I Some S is P
Some S is not $\bar{P}$
Some P is S
Some P is not $\bar{S}$</td><td>O Some S is not P
Some S is $\bar{P}$
Some $\bar{P}$ is S
Some $\bar{P}$ is not $\bar{S}$</td><td>I' Some $\bar{S}$ is $\bar{P}$
Some $\bar{S}$ is not P
Some $\bar{P}$ is not S
Some $\bar{P}$ is $\bar{S}$</td><td>O' Some $\bar{S}$ is not $\bar{P}$
Some $\bar{S}$ is P
Some P is not S
Some P is $\bar{S}$</td></tr>
</table>

The table contains four groups of universal judgments and four of particular judgments. In each group there are two affirmative and two negative judgments, one with the subject $\bar{S}$ or S and one with the subject $\overline{P}$ or P. The groups of judgments correspond to the eight types of fundamental judgments, so the first judgment in each group is one of these judgments and is marked with the respective symbol.

From the analysis of the judgments of each group it is found that the second, third and fourth judgments are obtained from the first judgment through conversion and obversion.

Based on Table 4, the relationships between judgments will be determined.

a) Equivalence relationships of the judgments

Two judgments are in the equivalence relationships when they represent the disjunction of the same unique and well-determinate relations.

The definition says that the judgments contained in every groups of the Table 4 are equivalent to each other. Each judgment in a group is equivalent to each of the other three judgments of the same group, due to the symmetry of the equivalence relation. The equivalence of the judgments belonging to a group shows that the relations between the terms of each judgment, terms that vary from the judgment to the judgment within the group, are of the same form. Because of equivalence, each judgment in a group can be reduced, by conversion and obversion, to any of the other three judgments of the same group.

In order to obtain the equivalence of judgment groups, Florea Țuțugan applied conversion and obversion to the eight fundamental judgments, obtaining the same groups of judgments as those in Table 4.

b) Contradiction relationships of the judgments

Two judgments are in a contradiction relationship if they have no unique and well-determinate relation in common, and the sum of their relations is equal to all seven unique and well-determinate relations.

Applying the definition, it follows from Table 4, that each judgment of a group that contains universal judgments has as contradictory judgments, due to equivalence, all judgments of the particular group that do not contain the disjunction of the considered universals, and the sum of the disjunctions of the two groups is equal to the seven unique and well-determinate relations. Thus, for example, each universal judgment from the first group of universal judgments (A-group) is contradictory to all particular judgments from the second group of particular judgments (O-group). As the relation of contradiction is symmetrical, each particular judgment from the second group (O-group) is contradictory to all universal judgment from the first group (A-group). The results are in line with Aristotle's statements from *On Interpretation* (10, 20a, 27-29) "the sentence «Some people are non-right» follows from the sentence «Some people are not right» which is opposite to «Every man is right»". Since the phrase "follows from" has the meaning "is equivalent to", the universal judgment has as contradictory the two equivalent particular judgments. Because the relation of contradiction is symmetrical, the two individuals have as contradictory the same universal.

As Florea Țuțugan considered only the eight fundamental judgments, he gave the following contradiction relationships:

A-O; E-I; A'-O'; E'-I'

specifying that they are symmetrical.

Considering also the negation operation, the particular judgments are the denial of universal judgments of opposite quality, and vice-versa, universal judgments are the denial of particular judgments of the opposite quality. Consequently, the knowledge of one also determines the knowledge of the other. So, universal judgments are equivalent to the denial of particular judgments of opposite quality, and vice versa, particular judgments are equivalent to the denial of universal judgments of opposite quality. Due to these properties, the 32 judgments can be divided into four groups of equivalent judgments, each group having eight judgments of which four universal and four particular, but of the opposite quality. These groups are made up of judgments corresponding to the association of the above-mentioned fundamental judgments.

c) *Subordination (subalternation) of the judgments*

A judgment is found in a relationship of subordination to another judgment if the first judgment includes in addition to the relations of the second judgment at least one unique and well-determinate relation.

From the analysis of the judgment disjunctions, it is found that the disjunction of each group of universal judgments is part of two disjunctions of particular judgments. As a result, two universal judgments in a group have subordinates (subalterns) in one of the two groups of particular judgments, and the other two universal judgments have their subordinates (subalterns) in the second group of particular judgments. For example, each of the first two universal judgments from the first group of universal judgment (A-group) has as subaltern, in the same order, one of the first two particular judgments from the first group of particular judgments (I-group), and the last two universal judgments of the first group of universal judgments (A-group) have as subaltern the last two judgments from the third group of particular judgments (I'-group). The disjunctions of the two groups of particular judgments have in their composition the disjunction I1 V II1 of the first group of universal judgments (A-group). Since the two particular judgments in a group, although of different qualities, are equivalent, each universal judgment can be considered to have two subalterns. They agree with the results of Florea Țuțugan (*op. cit*. p. 23):

A-I, I';   A'-I, I';   E-O, O';   E'-O, O'

d) Superordination (superalternation) relationships of the judgments

A judgment is in a relationship of superordination to another judgment if the second judgment includes, besides the relations of the first judgment, a unique and well-determinate relation in addition. These relationships are the inverse of subordination relationships, that is, each particular judgment has as superalterns two universal judgments.

e) Contrariety relationships of the judgments

Two judgments are in contrariety relationships if they have in common no unique and well-determinate relation, and their sum is not equal to the seven unique and well-determinate relations. The contrariety characterizes the universal judgments of the opposite quality. Investigating the disjunctions of the universal judgment groups from Table 4 it has been found that for each group there are two other groups with which it has no common relation. Consequently, two of the judgments of each group have the contraries in one of the two groups, and the other two judgments in the second group with which it has no unique and well-determinate common relation. For example, it is considered the first group of universal judgments (A-group), in which the first two judgments have as contraries the first two judgments of the second group (E-group), and the last two universals of the first group (A-group) are contraries to the last two judgments of the fourth group of universal judgments (E'-group). However, due to the equivalence of the judgments in a group, it can be considered that each judgment in a group is contrary to all the universal judgments of the other group. This property also includes symmetry. These are consistent with the results of Florea Țuțugan (*Ibidem* p. 25), with the specification that he limited himself only to the fundamental judgments (A, A', E and E').

f) Subcontrariety relationships of the judgments

Subcontrariety characterizes particular judgments. Two judgments are found in a relationship of subcontrariety if they share at least one unique and well-determinate relation and each has at least one unique and well-determinate relation in addition to the other.

Applying the above definition to the four groups of particular judgments it is deduced that the first group of particular judgments has as subcontraries the second and the fourth groups. Therefore, the first two judgments of the first group of particular judgments (I-group) have as subcontraries the first two judgments from the second group (O-group), and the two following judgments from the first group (I-group) have as subcontraries the last two judgments from the fourth group of particular judgments (O'-group).

The third group of particular judgments (I'-group) contains the subcontraries of the other judgments from the second and fourth particular judgments. The first two judgments of the third group (I'-group) are the subcontraries of the first two judgments from the fourth group (O'-group), and the next two judgments from the third group (I'-group) have as subcontraries the last two judgments of the second group (O-group).

As in the case of the contrariety relationship, due to the equivalence between the particular judgments of a group and the symmetry of subcontrariety, each particular judgment is subcontrary of the other particular judgments of the respective groups.

These coincide with Florea Țuțugan's assertion (*Ibidem* p. 25) that there are the following groups of subcontrariety:

I - O; I-O'; I'-O; I'-O'

g) Crossing (implicative indifference) relationships of the judgments

The relationship between judgments is of implicative indifference when, reciprocally, the truth or falsity of a judgment does not imply neither the truth nor the falsity of the other judgment.

Two judgments are in a crossing relationship or implicative indifference if they share at least one unique and well-determinate relation and each has at least one relation in addition to the other, but their sum does not equal to the seven unique and well-determinate relations.

From Table 4, by applying the definition, the following eight pairs of groups of judgments are identified in terms of implicative indifference:

1. Universal judgments
   – the first group (A) with the third group (A')
   – the second group (E) with the fourth group (E')
2. Universal judgments with particular judgments (written in this order)
   – the first group (A) with the fourth group (O')
   – the second group (E) with the third group (I')
   – the third group (A') with the second group (O)
   – the fourth group (E') with the first group (I)
3. Particular judgments
   – the first group (I) with the third group (I')
   – the second group (O) with the fourth group (O')

For all these relationships existing between the 32 judgments that can be obtained with two terms, both positive and negative, Florea Țuțugan gave several representations, of which it was chosen that similar to "Boethius' opposition square", with the difference that all the judgments corresponding to the fundamental ones written in the figure 1 are at the vertices of the squares.

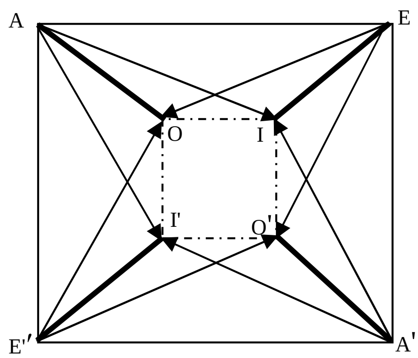


Figure 1

In Figure 1 the sides of the outer square signify contrariety relations; the sides of the inner square indicate subcontrariety relationships; the lines joining the vertices of the two squares indicate the contradictory relationships; the arrows mean the relationships of subordination (subalternation) or superordination (superalternation) respectively; the implicative indifference relationships were not shown because the figure is full.

Since the eight judgments written on the vertices of the squares are the representatives of the eight groups of equivalent judgments, it is to be understood that there are four judgments in each vertex.

VI. Conclusions

Between the extensions of two positive and negative terms, there are seven unique and well-determinate relations (identity, contradiction, subordination, superordination, contrariety, subcontrariety, and crossing) that generate 32 simple and distinct, universal and particular judgments; the number of universal judgments is equal to that of particular judgments.

Any judgment is represented by a disjunction that depends on the type of judgment: in the case of the universal judgments it consists of two unique and well-determinate relations, and in the case of particular judgments it is made up of five unique and well-determinate relations.

For each category of universal or particular judgment, four separate disjunctions are obtained, which divided every category into four groups of four judgments (Table 4); each group is represented by its own disjunction.

This division of judgments allowed: 1) to determine the relationships between judgments, which are of equivalence, contradiction, subordination (subalternation), superordination (superalternation), contrariety, subcontrariety and crossing; 2) proves the validity of the theory of Aristotelian conversion and the theory of immediate inferences.

The method used in this paper for determining the relationships between the predication judgments, by enunciating all the judgments that characterize each unique and well-determinate relation existing between two positive and negative terms, has allowed the determination of the disjunctions of all these judgments. It has led to the same results as those of Florea Țuțugan, who used the conversion and obversion of the eight fundamental judgments and who did not explain how he established the disjunctions that characterize these judgments. Also, the method used in this paper allows to verify Aristotle's theory of the judgment conversion and leads to its extension when the two terms of a judgment are positive and negative.

The fact that the two methods have achieved the same results proves their equivalence.

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