COMMENTS ON SAUL KRIPEK’S
A PUZZLE ABOUT TIME AND THOUGHT

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Abstract: The paper discusses an original and ingenious puzzle about time and thought due to Saul Kripke. Its connections with related issues and puzzles such as Russell’s paradox and Kaplan’s puzzle are addressed and explored. The problems raised by the puzzle are dealt with briefly, emphasizing the comparative worth of Kripke’s findings in terms of simplicity and modesty of its assumptions. Then the attempted solutions are analyzed and explored, the most important and promising avenue for future work being the extension of the apparatus developed by Kripke in his groundbreaking paper on truth to intensional logic.

Keywords: puzzle about time and thought, possible world, proposition, Kaplan’s puzzle about the cardinality of possible worlds, Russell’s ramified theory of types, truth, partial interpretation.

I. THE PUZZLE

This short paper upon which I am commenting is vintage Kripke. Fully packed with startling ideas and profound background connections, disarmingly simple in its statement, but nevertheless surprising and rich in its intuitions and suggestions, this paper throws new lights on the ever intriguing notion of paradox.

The puzzle is very simple. Its assumptions are unexceptional. It raises deep and intriguing issues. Here it is: “[…] at a given moment I think of a set S of instants of time (call these instants ‘times’). […] I need not know whether the set in question is empty or not – I can think of it by a defining property.”

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Kripke’s Puzzle about Time and Thought:

I think, at a certain time \( t_0 \) of the set \( S_0 \) where \( S_0 \) contains all times \( t \) at which I am thinking of a given set \( S_t \) of times, and \( S_t \) does not include \( t \) itself. Symbolically:

\[ S_0 = \{ t \mid S_t \text{ exists} \& t \not\in S_t \} \]

I am thinking of \( S_0 \) at a certain time \( t_0 \). Is \( t_0 \) a member of \( S_0 \) or not?

Suppose it is. Then this instant has the defining property of the set and hence it is one of those \( t \)s which does not belong to \( S_t \) and therefore neither to \( S_0 \). Then, maybe \( t_0 \not\in S_0 \). In that case, again, \( t_0 \) has the defining property for membership to \( S_0 \). Paradox ensues.

II. CONNECTIONS WITH RELATED ISSUES AND PUZZLES

(i) There is a similarity between Kripke’s Puzzle and Russell’s paradox. But to my own understanding, the stark contrast between them is more telling. For in Kripke’s puzzle the predicate we deal with is a perfectly meaningful and unexceptional notion, viz. „I think of a set \( S \) (of times)“. Nothing here is incompatible with Zermelo set theory or the like. We are just considering a subset of the set of all times, defined by the axiom of separation.

(ii) Another puzzle related to the issues which are raised by Kripke’s puzzle is one puzzle that David Kaplan discovered afterwards. What is this about? Prima facie it is a puzzle involving a cardinality issue that questions the notions of possible world and of proposition (equated with sets of worlds).

Here it is: „if the set of all worlds has a cardinality \( k \), the set of all propositions, taken to be the set of worlds, must have cardinality \( 2^k \). “ Kaplan makes the following assumption: for each proposition \( p \) and fixed time \( t_{0'} \) there is a world in which I entertain precisely that proposition \( p \) at \( t_{0'} \). This gives a one-to-one mapping of the power set of a set into the set itself. As we all know very well, this is ruled out by Cantor’s well-known theorem.

III. PROBLEMS RAISED BY THE PUZZLE

Is this Kaplanesque puzzle really raising a cardinality problem? Kripke’s diagnosis is very firm: Kaplan’s argument is not a cardinality problem for the notion of the set of all possible worlds, even though his argument appears to involve all those notions.

The main idea here is that Kripke’s puzzle is not about cardinality. And neither Kaplan’s puzzle is, for that matter. How is this working?

Kripke notices the well-known analogy between time and modality, tense logic and modal logic. Kripke’s puzzle is given for the case of time, and it is obvious that no one could legitimately raise doubts with respect to the

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1 Kripke [2011], p. 373.
meaningfulness of the notion of a set of all instants of time. Also the cardinality of the set of all instants of time is innocuous, being simply that of the continuum. All these make pretty clear that basically Kripke’s puzzle is not raising a cardinality issue.

We come thus at what might be called the comparative advantage of Kripke’s puzzle over Kaplan’s.

Kaplan’s puzzle has the disadvantage of misleading one into believing that due to the cardinality issue the notions of possible worlds and that of proposition, respectively, are deeply flawed. Now, Kripke has his own qualms about the notion of the set of all possible worlds. But his position has a net advantage over Kaplan’s. In what follows I will provide you with some reasons for believing this.

(1) The only assumption that Kripke’s puzzle makes is something truly unexceptional, viz. the subject is free to think of a set \( S_0 \) at a chosen time \( t_0 \).

(2) D. Lewis suggested that Kripke’s puzzle can be rephrased in terms of sets of people rather than of instants of time. To this Kripke adds the comment: „Presumably he [David Lewis] was thinking of egocentric logic, where people, in analogy to instants or worlds, are the appropriate indices.”\(^1\) This suggestion also reinforces the idea of there existing minimal and very modest assumptions behind the puzzle: I can just assume that „I am free to think at a designated time about the property of being a world \( w \) where I am thinking at that time about a set \( S_w \) of possible worlds \( w \) such that \( S_w \) does not have \( w \) as a member.”\(^2\)

(3) It is worth emphasizing that Kripke’s puzzle, in contradistinction to Kaplan’s, shows that there is no special problem about possible worlds that wouldn’t be a problem about times or people.

(4) And most important of all, the puzzling consequences of Kripke’s puzzle are not so damaging, after all, for there is nothing deeply disturbing paradoxical about its result in terms of instants of times. Here it seems to me that Kripke’s puzzle looks more benign and palatable that their would-be counterparts. In Kripke’s terms of instants of times, what the puzzle shows is that „most sets of times are not objects of my thought at any particular time“, whereas in (David Lewis’) terms of possible worlds its result is much more unpleasant since it seems to indicate that „there are propositions whose essence is such that no possible mind can entertain them.”\(^3\)

To sum up this section: the moral of Kripke’s puzzle’s compared to Kaplan’s puzzle is that Kaplan’s puzzle may obscure the essence of the matter, because his special paradox about possible worlds and cardinalities may give one false leads and impressions.

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\(^1\) Kripke [2011], p. 375.
\(^2\) Kripke [2011], p. 375.
\(^3\) Kripke [2011], p. 375.
IV. ATTEMPTED SOLUTIONS

a. The first natural option is to follow Russell in his ramified theory of types; this boils down to embracing a type-theoretic hierarchy of different predicates ‘thinking of.’ More specifically, by observing the type restrictions one has to accept that “the predicate $S_0$, as defined above, must be of a higher type than the properties defining the sets $S_i$ and the relation ‘thinking of’ involved in them.”

Kripke has long defended Russell’s ramified theory of types, due to the serious philosophical arguments that support it. According to Kripke, those include problems of intensional logic, illustrated by the liar paradox, of which Russell was fully aware.

b. But the most interesting and promising avenue for future work is Kripke’s suggested alternative to the previous solution. This grows out from his non-committal attitude toward proposing ramified type theory as the correct solution to his puzzle. Kripke’s proposed alternative is an intensional logic analogous to his own approach from “Outline of a Theory of Truth.” Kripke stops abruptly on this note saying that he does not “propose as of this writing to investigate the matter any further and “frame no hypotheses”.”

What Kripke suggests here very is to extend the apparatus developed in his groundbreaking paper on truth to intensional logic more generally, in line with the corresponding suggestion that he makes by the end of his “Outline of a Theory of Truth” that his approach be applied to languages containing modal operators. Of course, the inductive definition of the languages approximating to the minimal fixed point should be adjusted correspondingly.

The basic idea remains the same, though: the notion of groundedness is the key notion here because, according to the constructive procedure by which it is understood, not every sentence of the language considered here will be grounded. Some sentences, like the paradoxical sentence about time and thought considered in this paper here, may be ungrounded, not part of the extension of the candidate truth predicate, but also not part of its anti-extension either.

The well-known point is that the interpretation may be partial, some sentences characterized as ones to which the candidate truth predicate applies, others characterized as sentences to which the predicate does not apply, and others simply left uncharacterized. Keeping in line with Kripke’s suggestion that an ungrounded sentence does not clearly express a proposition, though there might be interpretations in which non-grounded sentences can express a proposition, one can diagnose the puzzling sentence in the paper saying that it is ungrounded and since it does not clearly express a proposition it is neither true nor false. Of course, this is only a sketch of the alternative

1 Kripke [2011], p. 376.
2 Kripke [2011], p. 377.
solution suggested by Kripke himself, and a fully detailed construction of the intensional language and of its truth predicate would be very illuminating as to the mechanics of the puzzle.

BIBLIOGRAPHY:

